Coleman & Rugg

Stresses in wire ropes bent over sheaves

Mechanical Engineering

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STRESSES IN WIRE ROPES BENT OVER SHEAVES

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THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

William Francis Coleman and Daniel Maltby Rugg

ENTITLED Stresses in Wire Ropes Bent Over Sheaves

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF Bachelor of Science in Mechanical Engineering

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BENDING STRESSES IN WIRE ROPES.

I. INTRODUCTION.

1. With the larger capacity cranes of the present day, it becomes a matter of some importance to determine the nature of the strains in the hoisting rope and the minimum size of sheave allowable for any given size or type of rope. Up to the present time all investigators have agreed that the stress due to bending is a large part of the total, but while all agree that the bending stress is large, their opinions differ widely as to its exact amount, or as to its percentage with reference to the whole.

In order to determine which of the theories already advanced was correct, or if they were all in error, an extensive series of tests were conducted at the University of Illinois under the auspices of the department of Mechanical Engineering. In these tests the object was to determine experimentally the bending stresses of ropes around sizes of sheaves most commonly used in practice. The ropes selected were 6x19 plow steel, and in this connection the writers wish to acknowledge the courtesy of The American Steel and Wire Co. in furnishing gratis, ropes and grips for carrying out the tests.

Before describing further these tests a resumé will be given of the theories and investigations which have been made along these lines up to the present time.

2. About 1865, Professor Reuleaux tried to solve the problem of bending stresses in the following manner. He took each wire as a simple beam in flexure and by the combinations of the formula $M=\frac{SI}{C}$ and $M=\frac{EI}{R}$ found that the stress in the extreme fiber due to bending was

$$S = \frac{Ed'}{D}$$

where

E = the modulus of elasticity

d'= the diameter of a single wire

D = the diameter of the drum

S = Stress in Lb. per sq. in. due to bending.

3. Mr. W. Hewitt in his book on "The Application of Wire Ropes to Transportation", published by The Trenton Iron Co., gives tables of the bending stresses on ropes with 19 wires and 7 wires to the strand as computed from his formula

$$K = \frac{Ea}{2.06\frac{R}{d}+C}$$

in which

K = bending stress in pounds

E = the modulus of elasticity

a = the area of metal in square inches

R = the radius of the bend in inches

d = the diameter of the individual wires in inches

C = a constant depending on the number of wires in the strand.

The values of d and C used in the computations are

7 wire rope

$d = \frac{1}{9}$ diameter of rope

$C = 9.27$

19 wire rope

$d = \frac{1}{15}$ diameter of rope

$C = 15.45$

4. The Hazard Mfg. Co. in their catalog on Wire Rope give the formula by Mr. E. T. Sederholm M.E.

$$S = 1{,}894{,}000\ \frac{d}{D}$$

where

d = diameter of rope in inches

D = diameter of drum in inches

S = stress per sq. in. due to bending.

5. In 1902 Josef Hrabak published his book entitled "Der Drahtseile", in which he enters into an elaborate investigation of the magnitude of the stress set up by the bending of wire ropes. Taking a wire rope in which the modulus of elasticity of the material of the wires is 28,500,000 and the angle of lay is 18°, he finds

$$f = \frac{.44Ed}{D}$$

i.e., the stress is .44 times that computed by the Reuleaux formula. This is derived from the equations

$$E = 0.36E_o$$

$$E' = E \sec^2 a. \sec^2 b$$

where E_o = the modulus of elasticity of the single wires.

E = the modulus of elasticity of the rope.

E' = the modulus of elasticity of the wires as laid in the rope.

6. In 1908 Mr. R. W. Chapman developed a formula by taking into consideration the angle of lay of the wires and of the strands in which he found that

$$S = .81 \frac{Ed}{D}$$

i.e., the stress is .81 times that computed by the Reuleaux formula.

7. The American Steel and Wire Co. publish tables giving the bending stresses for various sizes of ropes and sheaves. They are figured from the equation

$$S = \frac{Ed'}{D}$$

where E = 12,000,000 for plow steel rope.

8. About 1890 Andrew S. Biggart conducted a series of tests on the wear of ropes bent over sheaves. He tried various makes and types of ropes of one and three quarters inch circumference around sheaves from 5½" to 18¼". His method was to have a constant load of 1400 pounds on the rope and determine how many bends the rope would make before rupture. A curve showing his results graphically is given below.

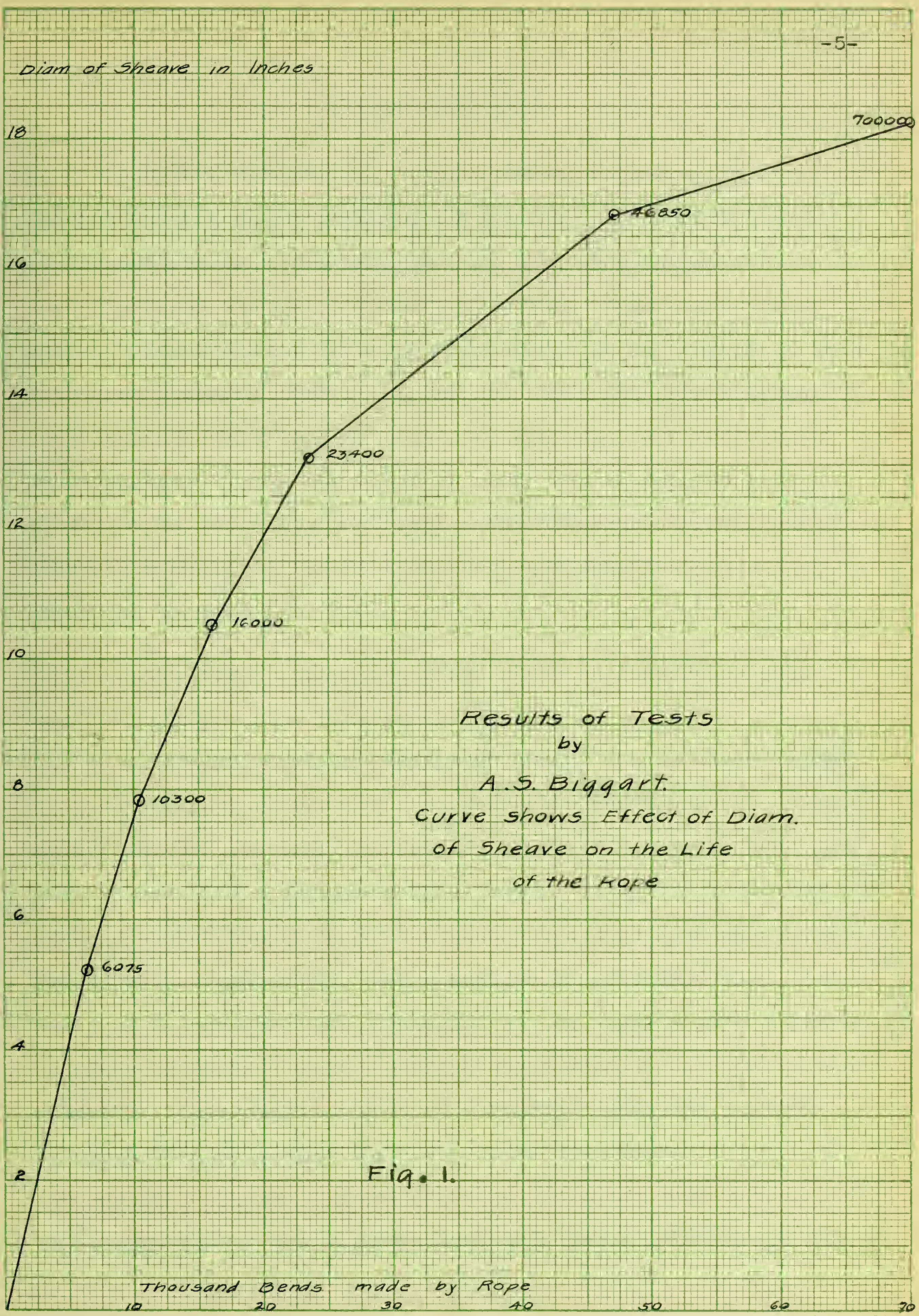

Diam of Sheave in Inches
18
16
14
12
10
8
6
4
2
700000
46850
23400
16000
10300
6075
Results of Tests
by
A. S. Biggart.
Curve shows Effect of Diam.
of Sheave on the Life
of the Rope
Thousand Bends made by Rope
10
20
30
40
50
60
70

Fig. 1.

II. METHOD OF TESTING.

9. GENERAL DESCRIPTION. The tests were made in the Laboratory of Applied Mechanics of the University of Illinois by the authors, as a thesis investigation under the direct supervision of Professor Herbert F. Moore. They consisted of static bending stress tests of three sizes of rope ($\frac{1}{2}$, $\frac{3}{4}$, 1 in.) over four sizes of sheaves (8, 12, 18, $22\frac{1}{4}$) and also a straight pull test for each size of rope. In this work all tests were in triplicate. Both the sizes of ropes and of sheaves were chosen as sizes in use in actual crane service. The $22\frac{1}{4}$ inch sheave was designed for 24 inches but through faulty moulding could not be made full size.

10. APPARATUS. In making these tests the apparatus used was that shown in the photograph Fig. 2.

(M) is a standard testing machine of 100000 pounds capacity made by the Philadelphia Machine Tool Co. of Philadelphia, Pa. This machine was of the usual two screw, power driven type, with a beam balance and platform on knife edges. In order to set the sheaves on the top of this machine the original head was removed and a new one made of 15 inch channels (C.C.) substituted. A piece of 4 inch shafting (S) milled at the ends to give a bearing surface of about 5 square inches, was seated on the top of the channels. This shaft carried at different times the several sizes of sheaves around which the ropes were bent.

(E) and (E') are the two extensometers used and show respectively the elongation of the straight rope and that portion which was

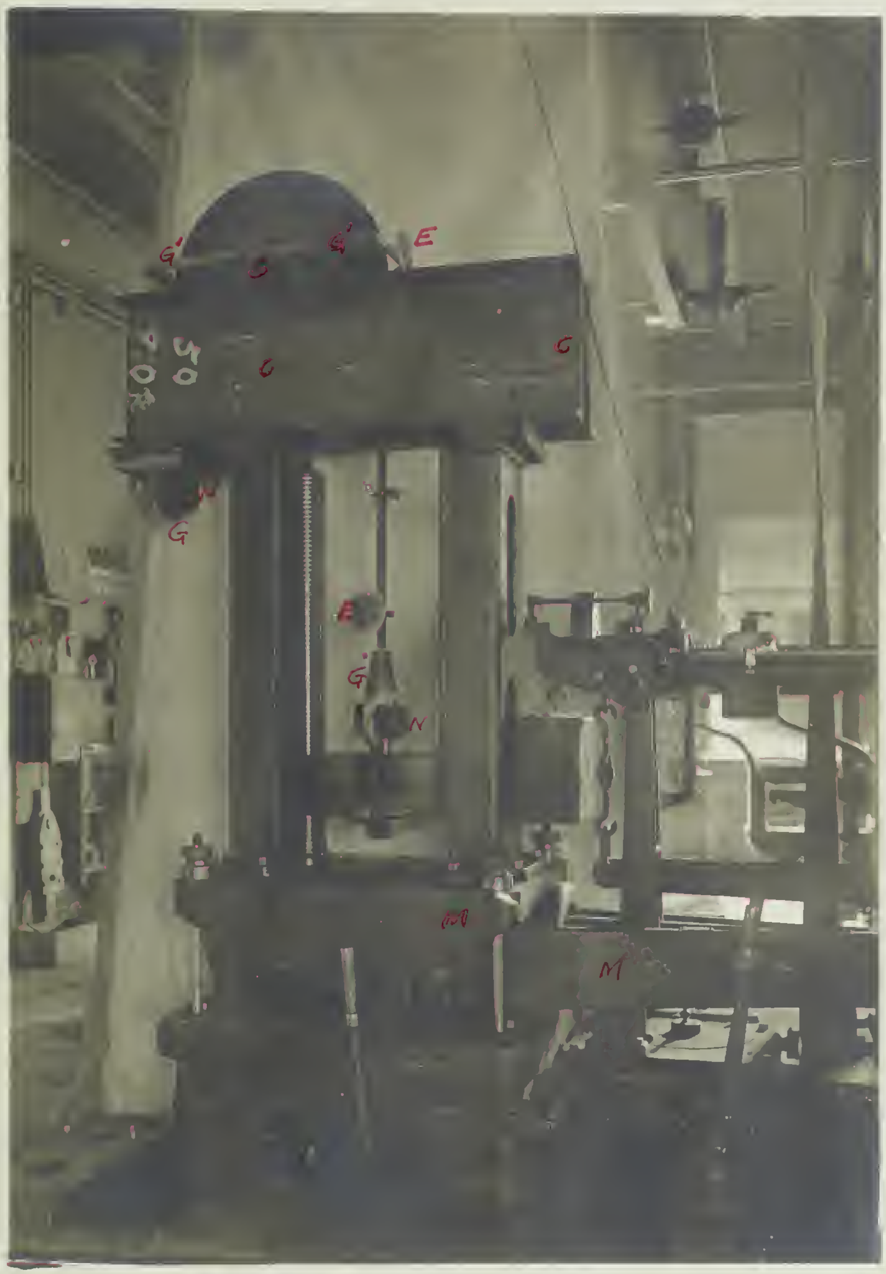


Fig. 2.

bent around the sheave. Readings from both were taken from a graduated dial on which a pointer revolved. The motion of the pointer was due directly to the elongation of the rope, but for convenience the elongation shown by the pointer was twice the actual amount, as the elongation measured was only relative at best.

GG are the grips attached to the rope by opening out the strands and babbitting the ends into comical shapes in the socket.

NN are nickel steel bars used to transmit the stress from the heads to the rope.

(G'G') is a carriage carring an extensometer. It is pivoted on the shaft and is used to measure the elongation over the sheaves.

11. METHOD. For the bending stress determination the rope was placed in the machine and the apparatus set up as shown in Fig. 2. The beam was balanced and a small initial load applied. This was taken at 500 lb. for the ½ inch rope, 1000 lb. for the ¾ inch rope and 2000 lb. for the 1 inch rope. The extensometer pointers were set to read zero at the initial loading. Readings were taken of load and deflection on each instrument at convenient intervals on the straight pull dial. This was done to a point within a few thousand pounds of rupture and then the instruments were taken off. The load was again applied and the rope carried to rupture noting the load at which the first wire snapped and the load at rupture. After rupture an examination was made of the rope, for number of strands breaking, place of break, and intensity of nicking of the wires.

The straight pull tests were conducted in the same manner with the exception that there was only one extensometer. Besides these two types of general tests a series of straight pull tests were made of single wires of each size from each size rope. This was done to find the yield point and ultimate strength of the single wires and to try and determine the modulus of elasticity.

All data was recorded on the standard log book of the department and curves of each test were drawn and they appear complete in Part III.

Fig. 3

Fig. 4.

III. DATA.

12. The contents of this part is divided into four general divisions: first, general data; second, curves showing the results of each separate test; third tables summarizing these tests and of bending stresses for test conditions as computed from various formulas and fourth curves showing graphically the relation between the formulas and tests.

13. GENERAL DATA.

All the ropes showed severe nicking at the rupture points. See Fig. 4. In all cases several wires snapped before the rope gave way as a whole. Four was the greatest number of strands which broke on any specimen. See Fig. 3 for characteristic break. The strands which ruptured were always next to the sheave at the rupture point. On a rope which was stressed until over a dozen wires snapped and the load then released and the strands opened up, the inside wires were intact and all of the wires which ruptured were outside wires and failed, due to nicking where the strands gouged into each other.

TABLE I.

RUPTURE OF SINGLE WIRES

Diam. of Rope	Diam. of Wire	Test No. 1	Test No. 2	Test No. 3	Av.	Stress Pounds per Square Inch	Area of Metal in rope Sq. Inch
½	.029	130	145	140	138	(209000	.1169
	.039	240	210	230	227	(190000	
¾	.044	325	330	315	323	(212000	.2563
	.055	550	580	600	576	(242500	
	.060	646	690		678	(240000	
1	.056	610	630	630	623	(253500	.4197
	.072	970	980	975	975	(239600	
	.075	1030	1050	1050	1044	(237000	

TABLE II.

LOAD AT RUPTURE IN POUNDS

Diam. of Rope	Test No.	Diameter of Sheaves in Inches 8	12	18	22¼	Straight Pull
	1	18500	19600	19490		20870
½	2	18350	19490	19870		20550
	3	-----	19350	19840		19600
	Av.	18425	19470	19730		~~29340~~ 20340
	1	44820	44210*	47720	48650	50340
¾	2	43080*	46100	47850	48120	50710
	3	44740	46230	47530	48620	50470
	Av.	44780	46160	47700	48460	50510
	1		72380	74420	60500*	85900
1	2		72180	72100	75040	85850
	3		71080	72280	73010	86130
	Av.		71880	72930	74025	85960

*Not considered

TABLE III.

LOAD AT BREAKING OF FIRST WIRE IN POUNDS

Diam. of Rope	Test No.	Diameter of Sheaves in Inches 8	12	18	22¼	Straight Pull
	1	17020	19470	19020		20240
½	2	18350	19160	19190		17750
	3	17310	14910	19780		18970
	Av.	17560	19315	19330		18990
	1	42890	28560	45820	45350	-----
¾	2	42980	44100	44630	47340	47160
	3	39740	36870	46890	42700	49700
	Av.	41870	39840	45780	45130	48430
	1		71740	72610		68250
1	2		70790	67750	67480	78730
	3		70930	69380	61000	79740
	Av.		71150	69115	64240	75575

TABLE IV.

STRESS AT RUPTURE IN POUNDS PER SQUARE INCH

Diam. of Rope	Diameter of Sheaves in Inches 8	12	18	22¼	
½"	158000	167000	169200		174000
¾"	173000	178000	184000	186800	195000
1"		173000	175800	178500	207200

TABLE V.

LOADS OF RUPTURE

	Diam. of Rope	Diameter of Sheaves in Inches: 8	12	15	18	21	22¼	24
Hewitt's	½	6940	11180	13000	14200	14930		15710
	¾	7910	21110	26210	30560	33310		35460
Formula	1	- 4640	23160	34960	42960	48860		53360
$\frac{Ed}{D}$	½	5740	10600	12560	13850	14750		15480
	¾	1960	18110	24610	29110	31990		34310
	1	-17790	16710	30560	39760	46360		51360
Hrabak's	½	13910	16060	16920	17580	17890		18200
	¾	29120	36260	39110	41110	42360		43390
Formula	1	40360	55560	61590	65660	68560		70760
U. of I.		18425	19470		19730			
Tests		44780	46160		47700		48460	
Av's.			71880		72930		74025	

TABLE VI.

BENDING STRESSES IN POUNDS.

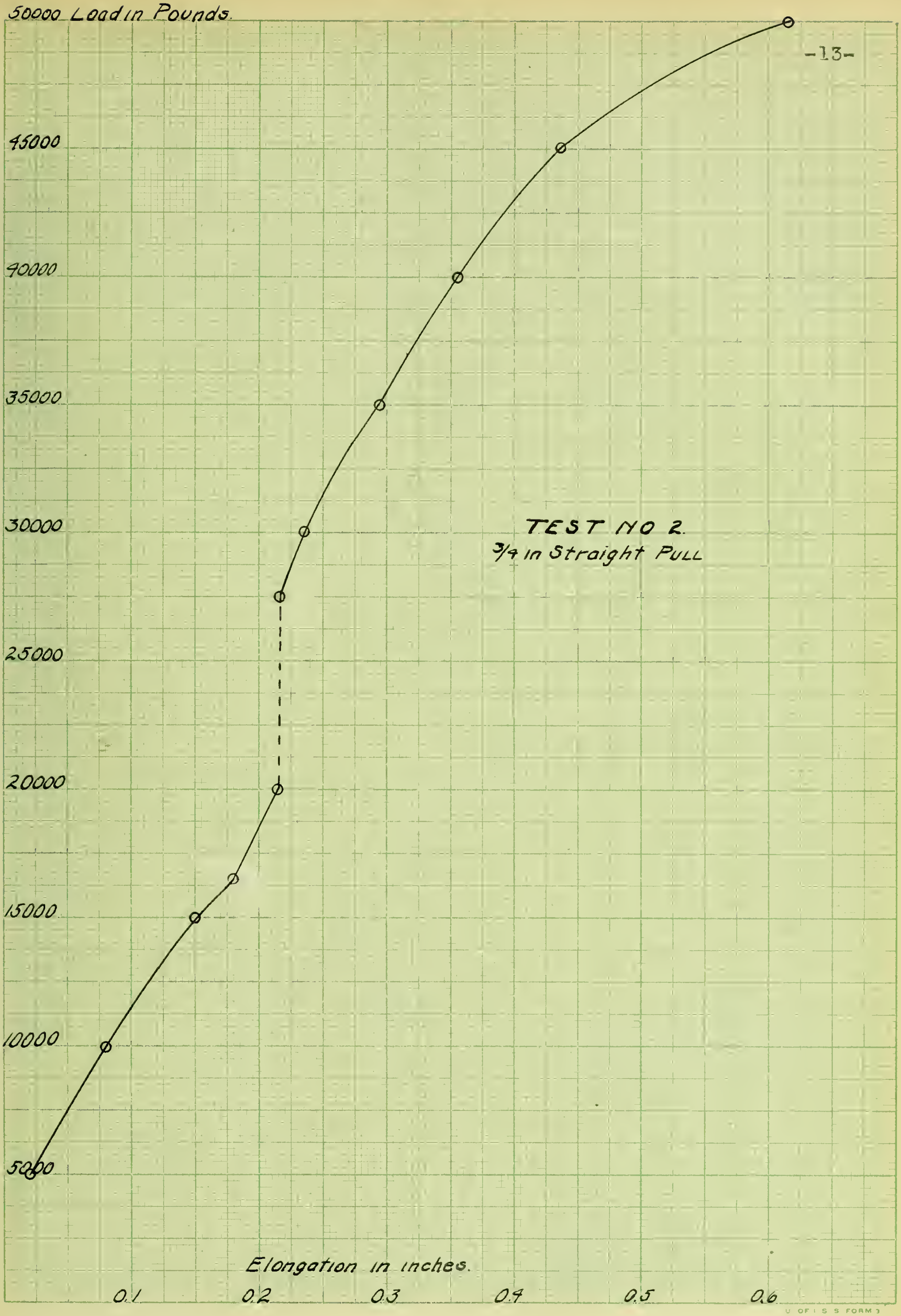
Formula of Test	Diam. of Rope	Diameter of Sheaves: 8	12	15	18	21	22¼	24	27	36
U. of I.	½	2445	1400		1140					
	¾	5690	4310		2770		2010			
Tests	1		14080		13030		11935			
Hewitt's	½	13400	9160	7340	6140	5410		4630		3160
	¾	42600	29400	24300	19950	17200		15050		10200
Formula	1	20600	62800	51000	43000	37100		32600		22100
Reu-leaux's	½	14600	9740	7780	6490	5560	5250	4860	4330	3240
	¾	48550	32400	25900	21400	18520	17500	16200	14400	10800
Formula	1	103750	69250	55400	46200	39600	37300	34600	30800	23050
Hrabak's	½	6430	4280	3420	2760	2450	2310	2140	1900	1440
	¾	21390	14250	11400	9400	8150	7700	7120	6340	4750
Formula	1	45600	30400	24370	20300	17400	16400	15200	13550	10120

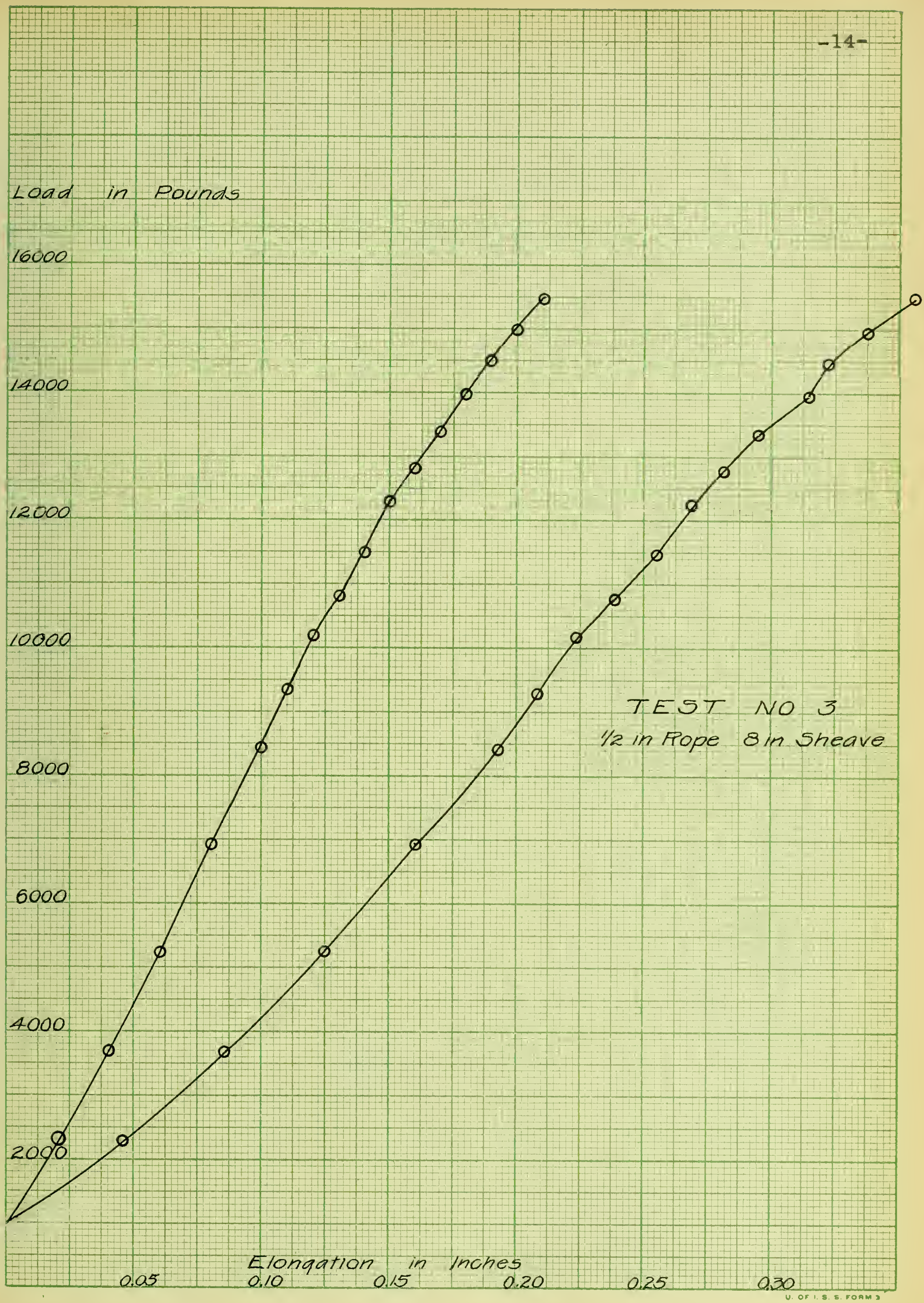
CURVES SHOWING GRAPHICALLY

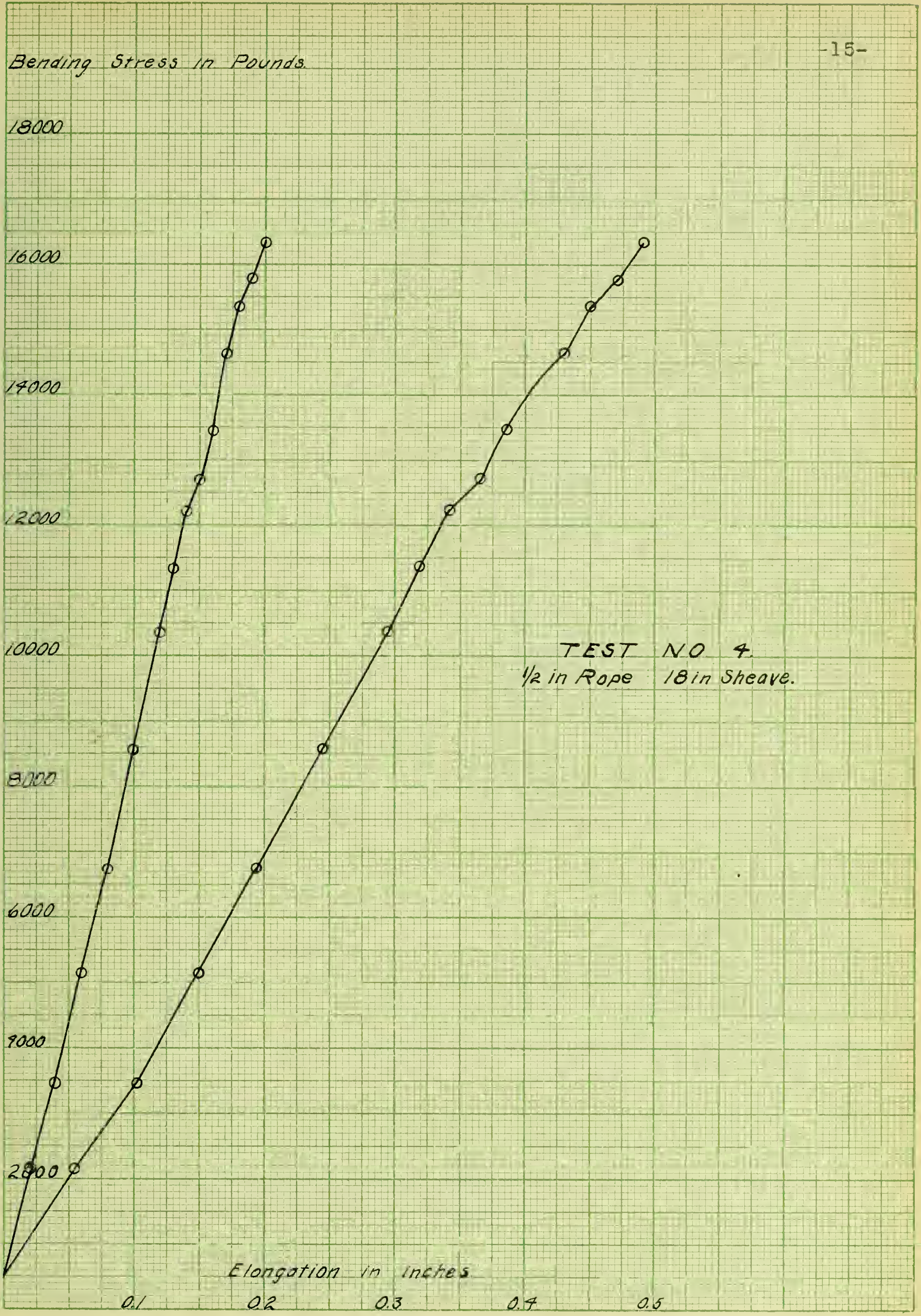
THE RESULTS

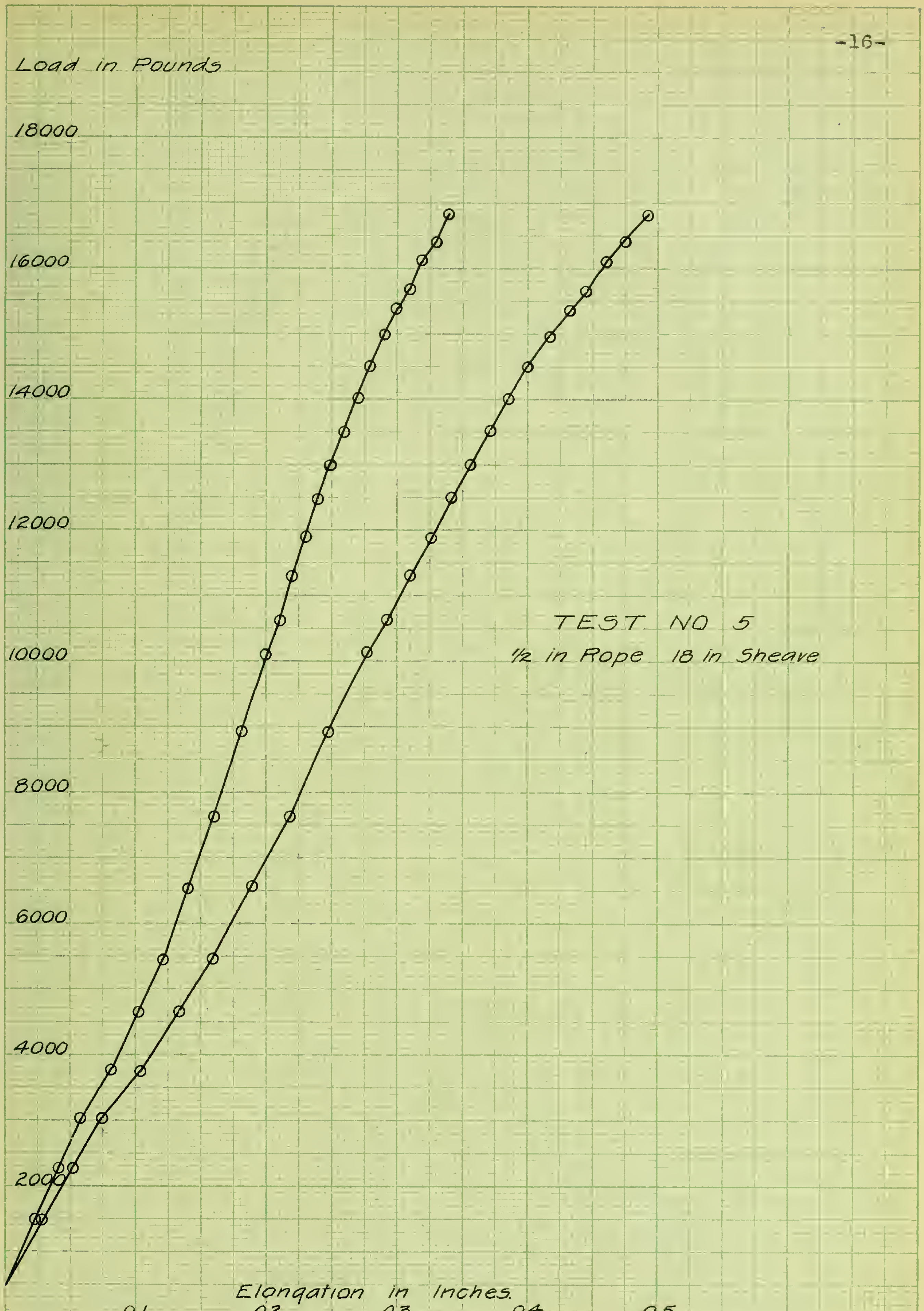
OF THE

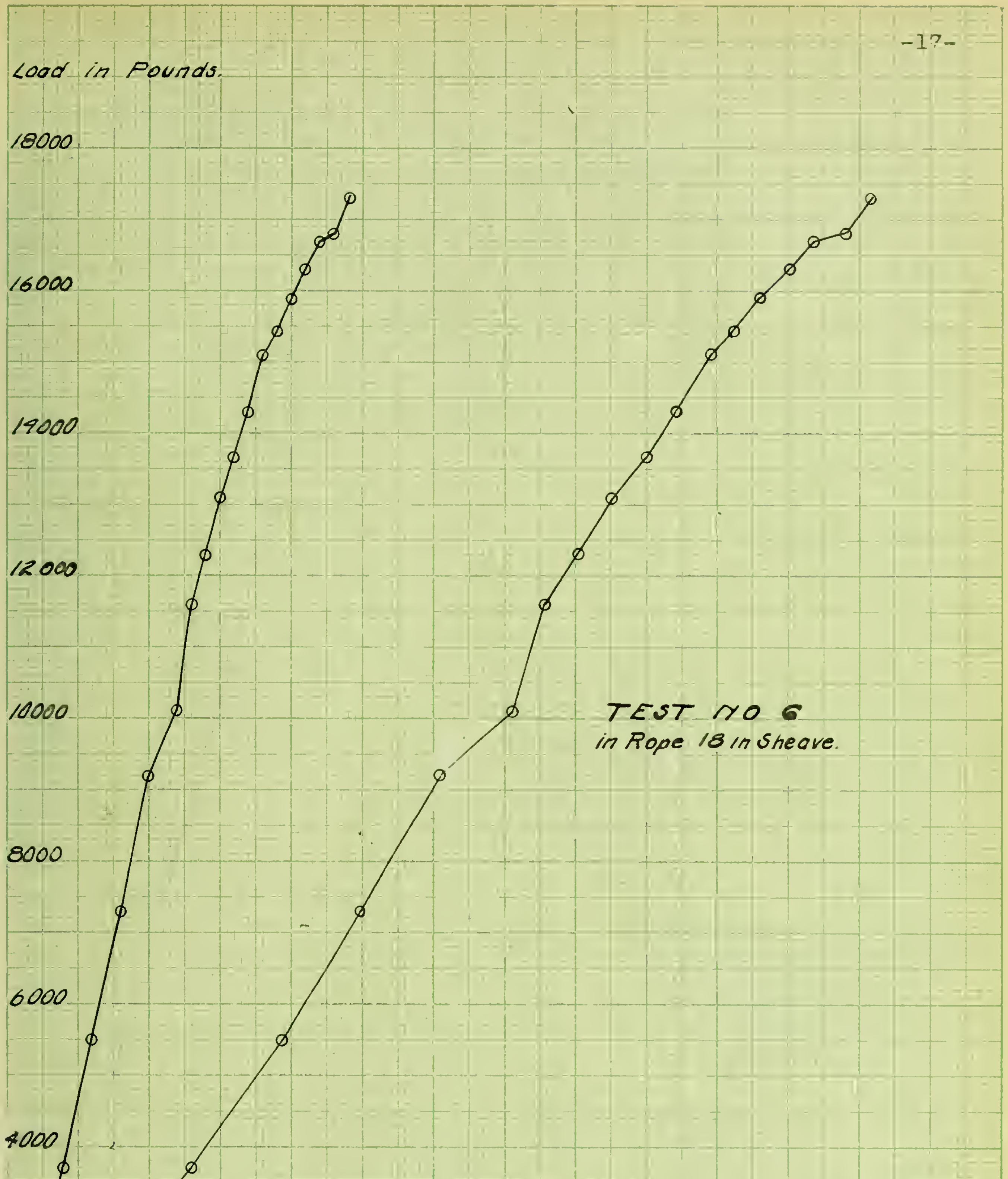
INDIVIDUAL TESTS

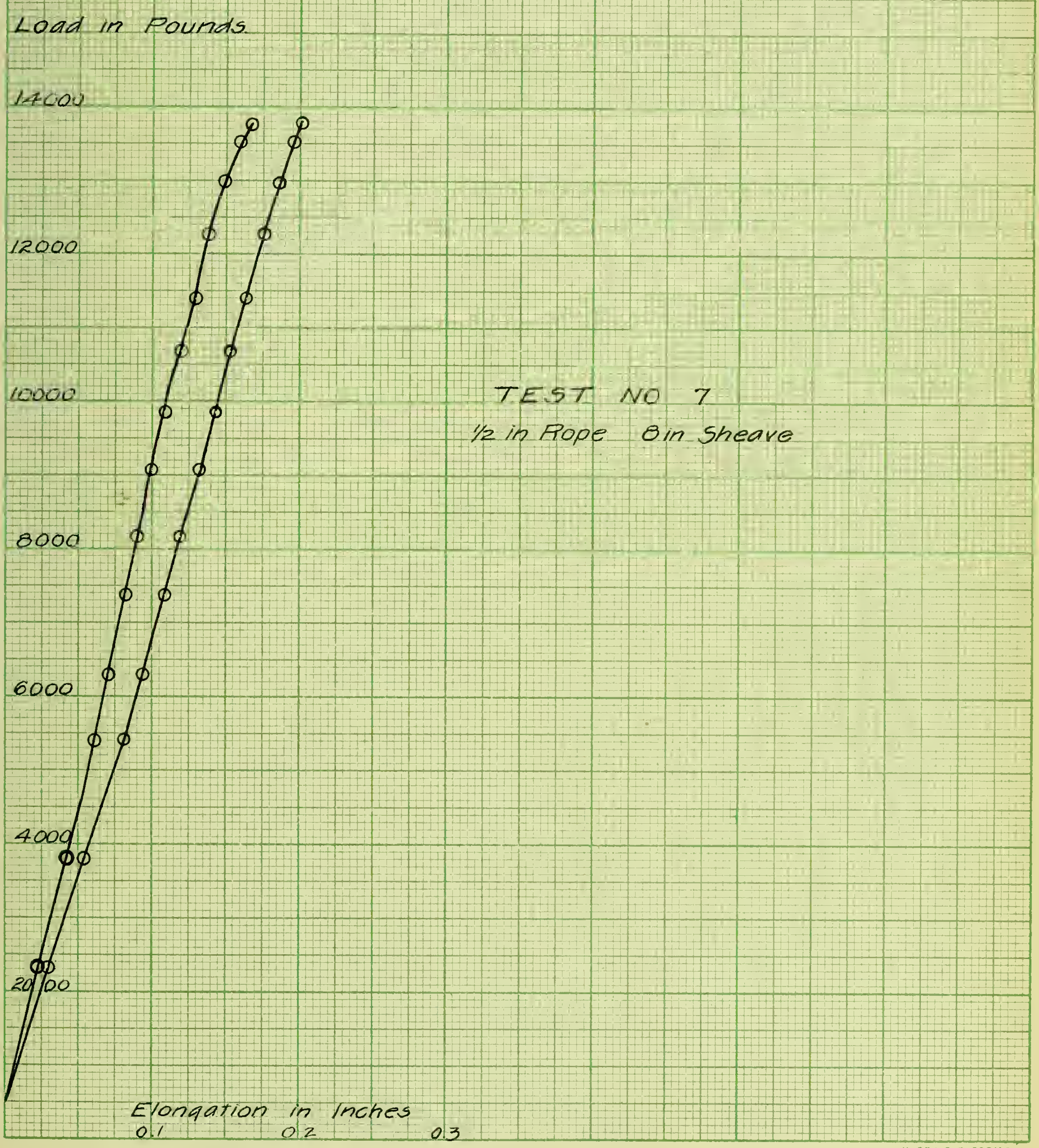

50000 Load in Pounds.
45000
40000
35000
30000
25000
20000
15000
10000
5000
TEST NO 2.
3/4 in Straight Pull
Elongation in inches.
0.1
0.2
0.3
0.4
0.5
0.6
U OF I S S FORM 3

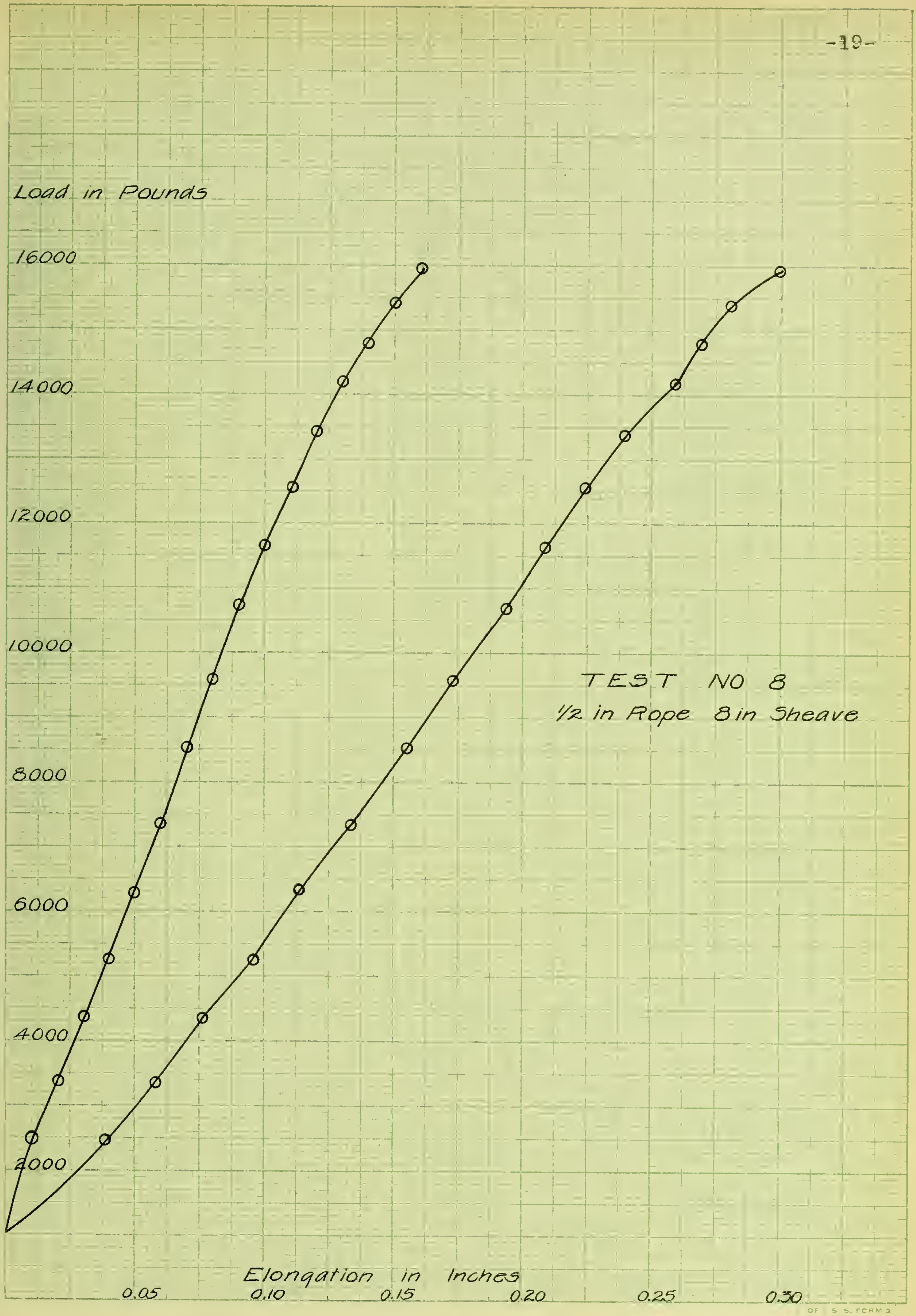

Load in Pounds
16000
14000
12000
10000
8000
6000
4000
2000
TEST NO 3
1/2 in Rope 8 in Sheave
Elongation in Inches
0.05
0.10
0.15
0.20
0.25
0.30
U. OF I. S. S. FORM 3

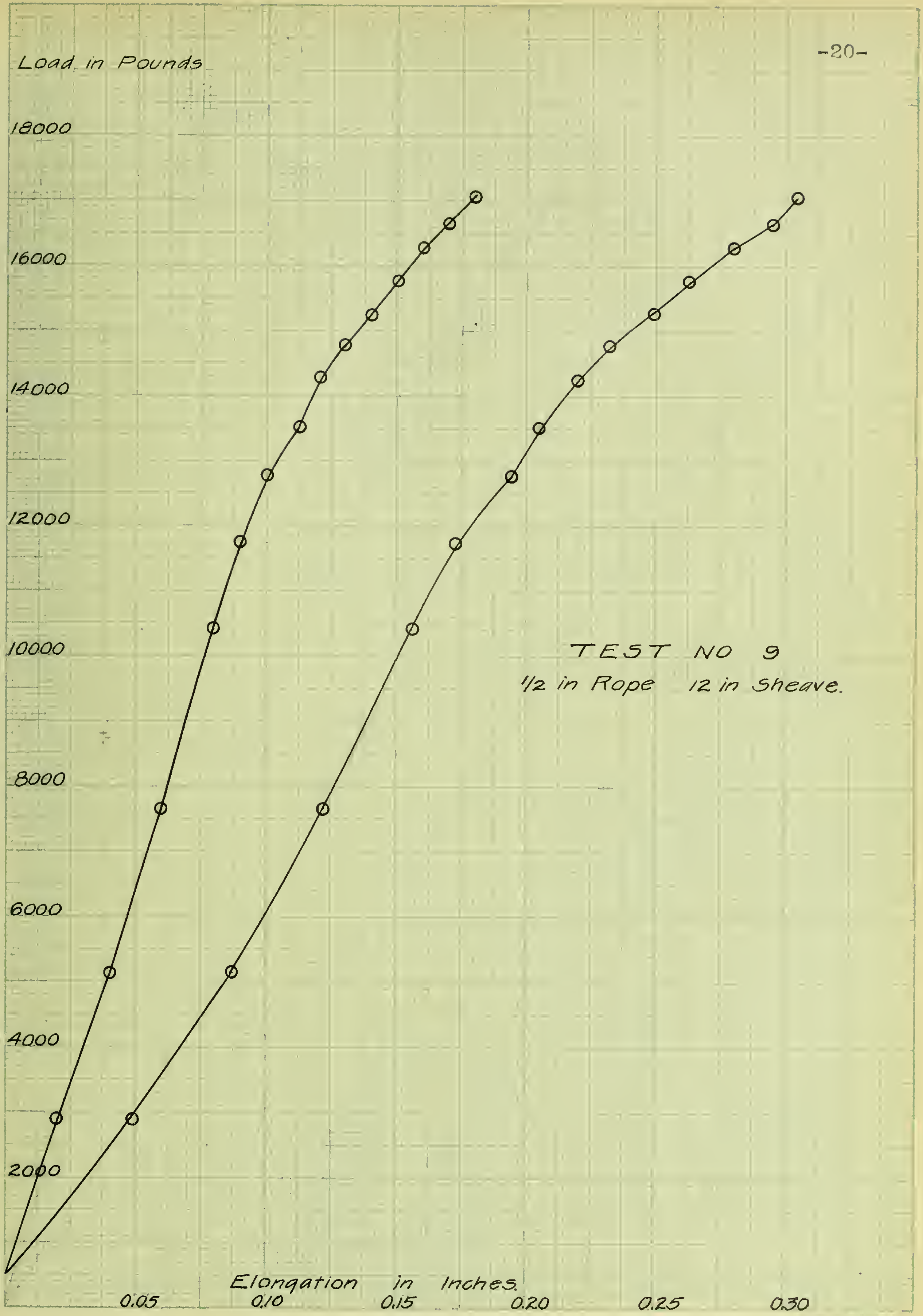

Bending Stress in Pounds.
18000
16000
14000
12000
10000
8000
6000
4000
2000
TEST NO 4.
1/2 in Rope 18 in Sheave.
Elongation in Inches
0.1
0.2
0.3
0.4
0.5

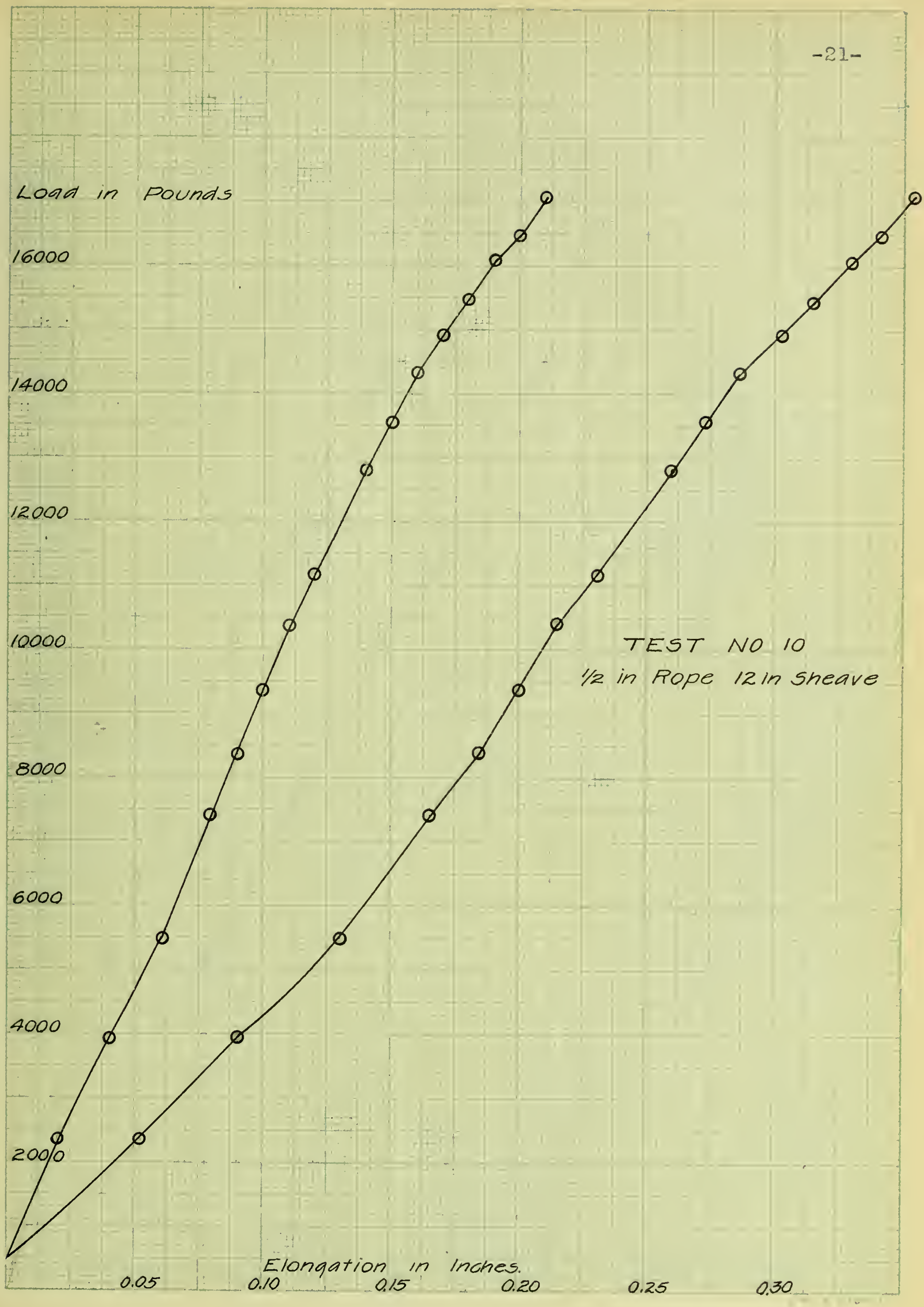

Load in Pounds
18000
16000
14000
12000
10000
8000
6000
4000
2000
TEST NO 5
½ in Rope 18 in Sheave
Elongation in Inches.
0.1
0.2
0.3
0.4
0.5

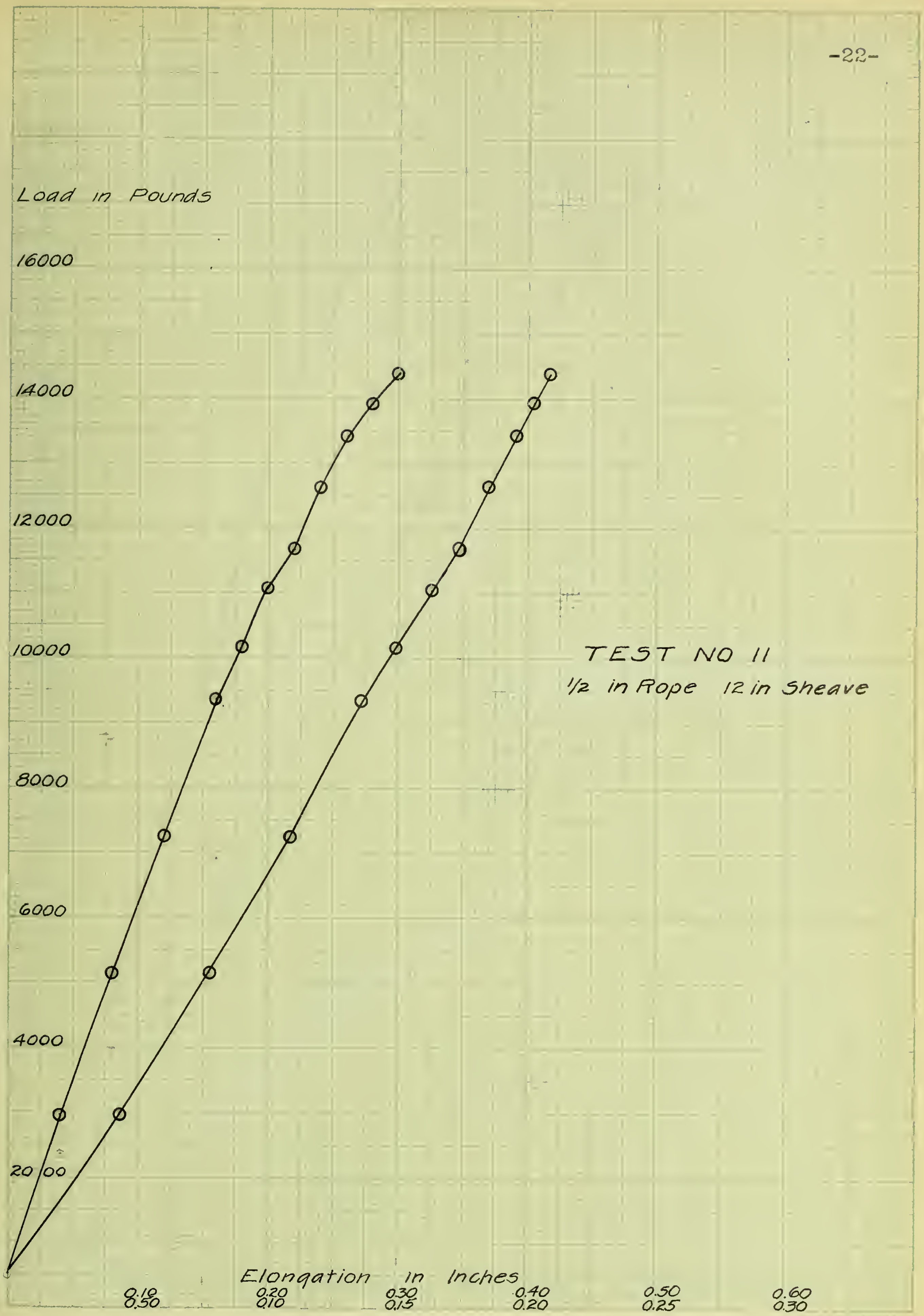

Load in Pounds.
18000
16000
14000
12000
10000
8000
6000
4000
2000
TEST NO 6
in Rope 18 in Sheave.
Elongation in inches.
0.1
0.2
0.3
0.4
0.5
0.6
0.7

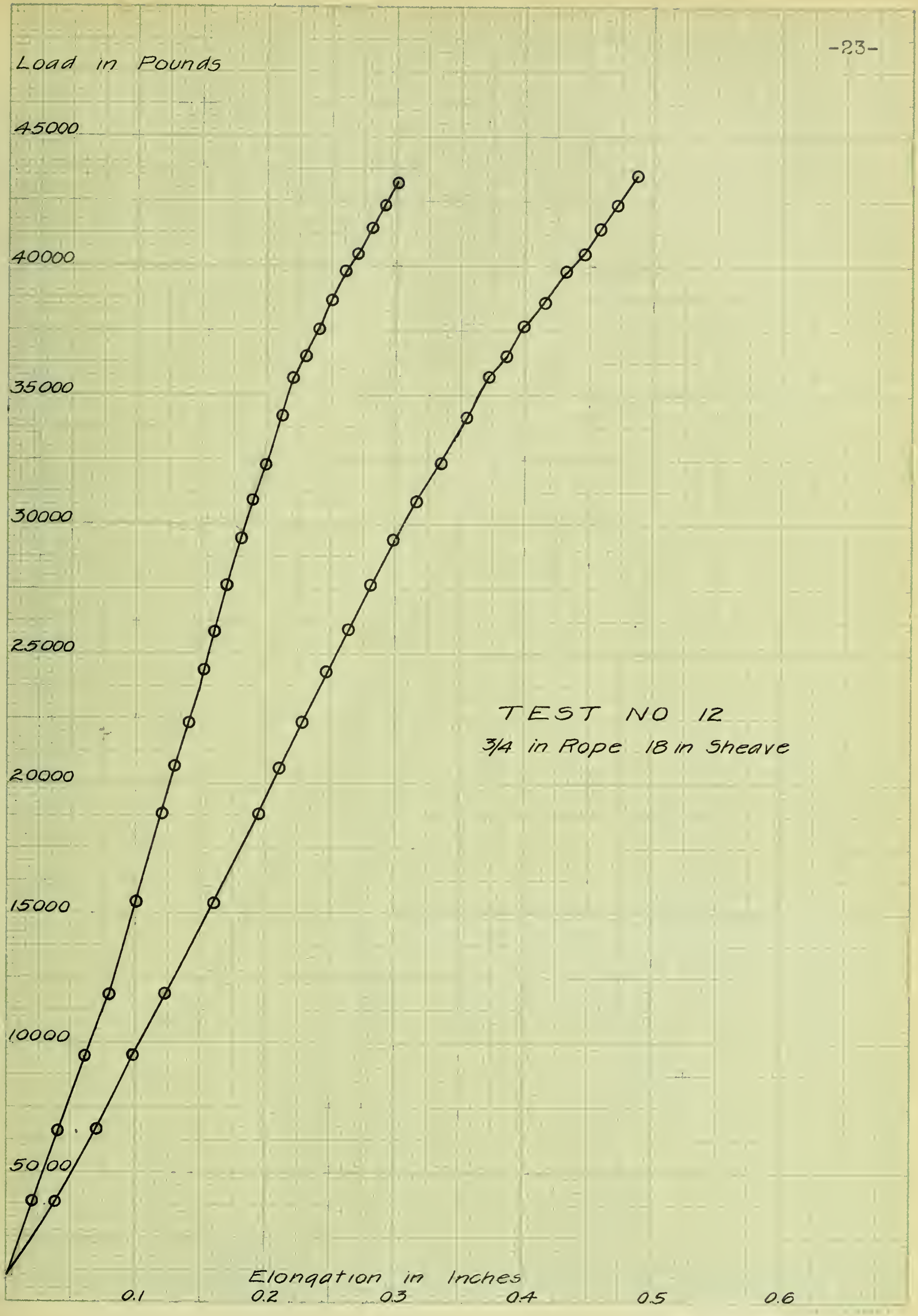

Load in Pounds
14000
12000
10000
8000
6000
4000
2000
TEST NO 7
½ in Rope 8 in Sheave
Elongation in Inches
0.1
0.2
0.3

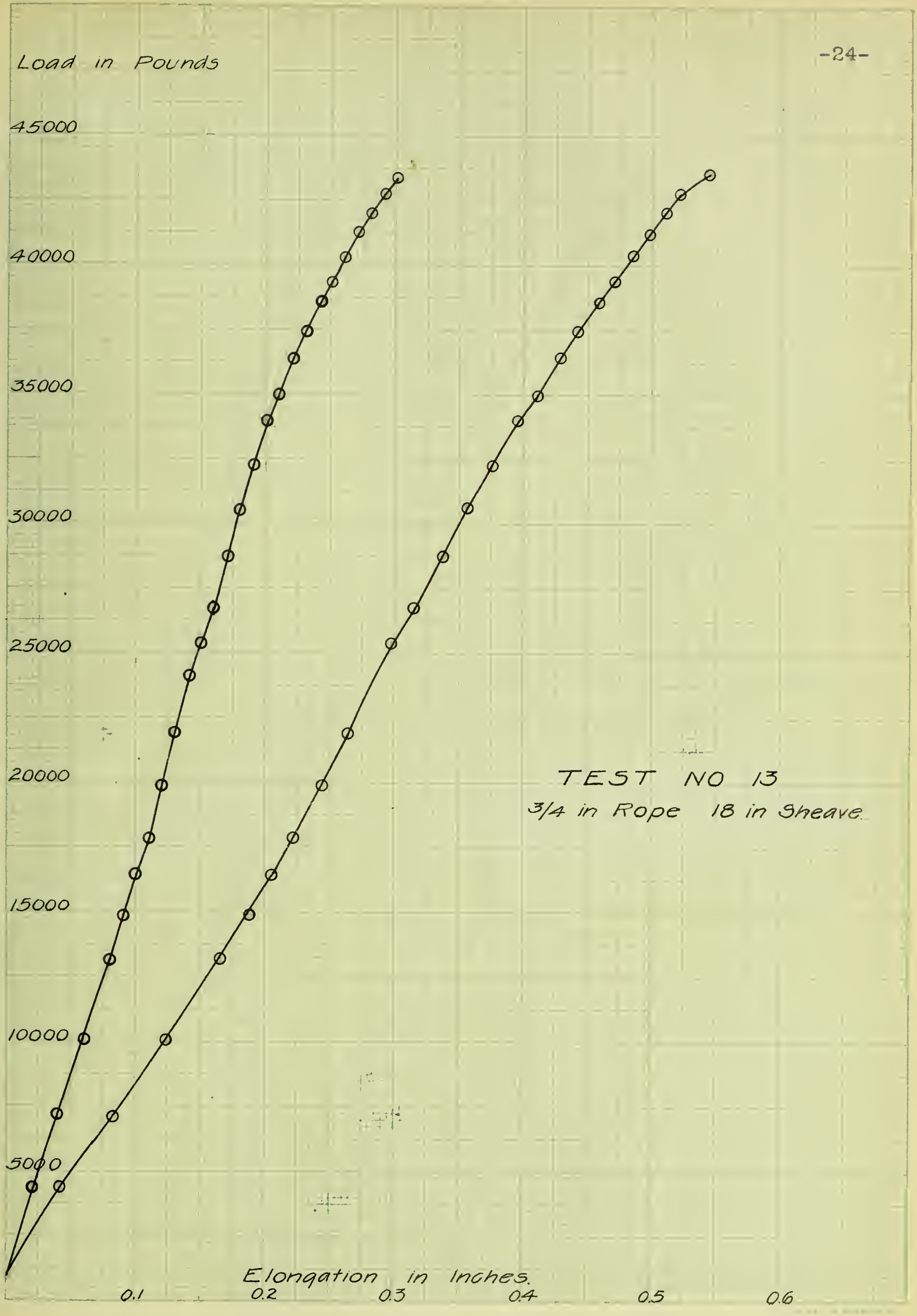

Load in Pounds
16000
14000
12000
10000
8000
6000
4000
2000
TEST NO 8
1/2 in Rope 8 in Sheave
Elongation in Inches
0.05
0.10
0.15
0.20
0.25
0.30

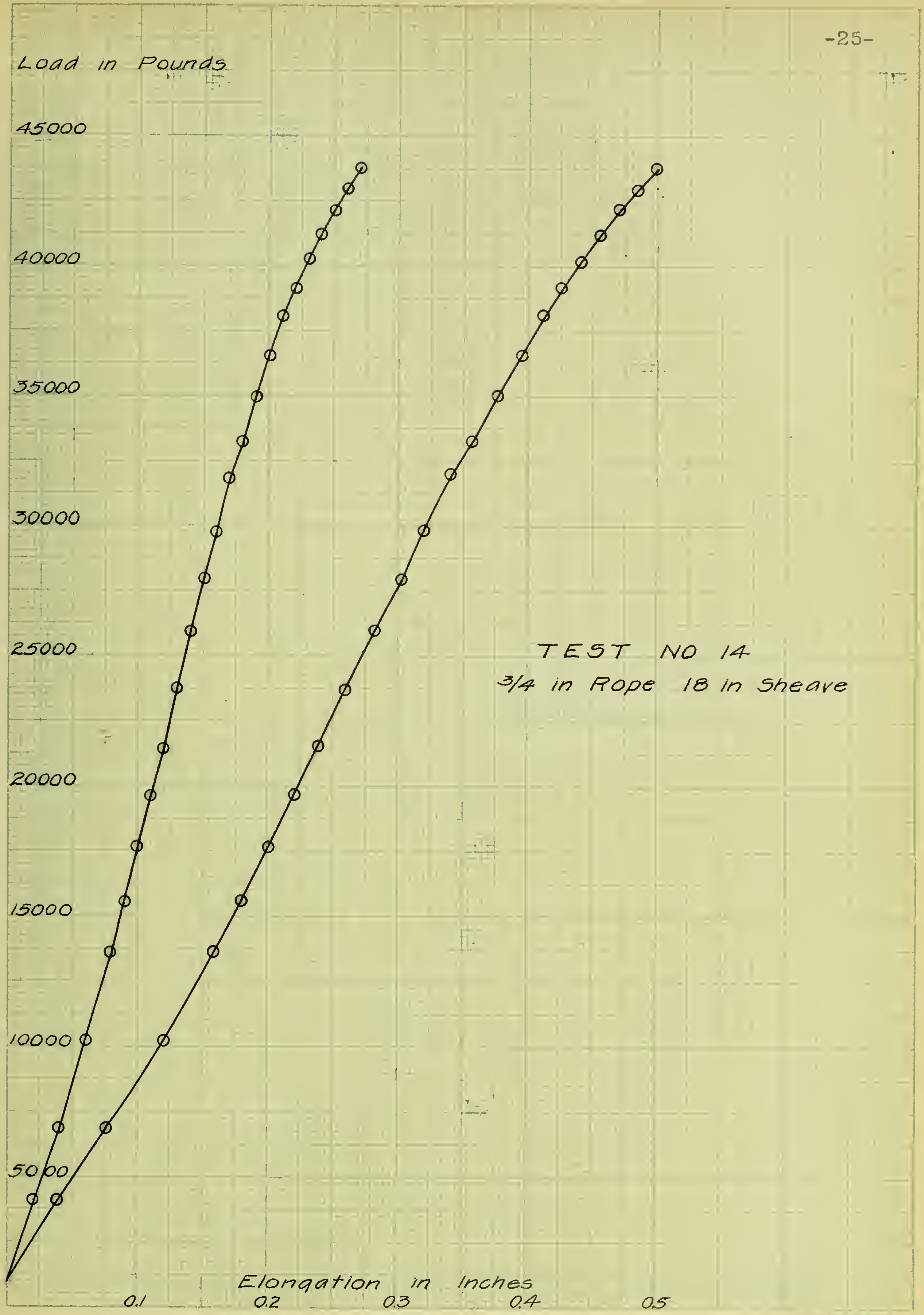

Load in Pounds
18000
16000
14000
12000
10000
8000
6000
4000
2000
TEST NO 9
1/2 in Rope 12 in Sheave.
Elongation in Inches.
0.05
0.10
0.15
0.20
0.25
0.30

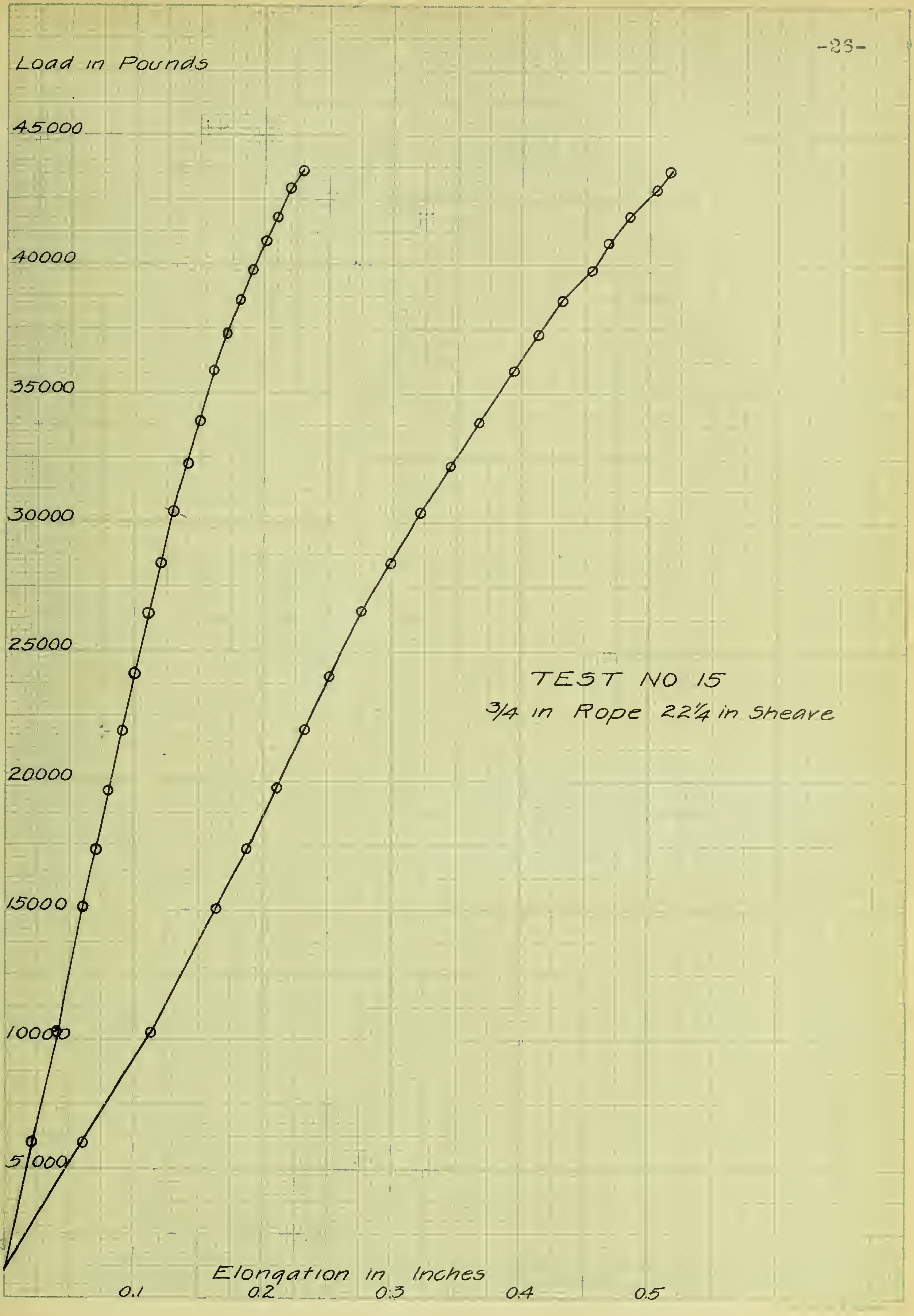

Load in Pounds
16000
14000
12000
10000
8000
6000
4000
2000
TEST NO 10
1/2 in Rope 12 in Sheave
Elongation in Inches.
0.05
0.10
0.15
0.20
0.25
0.30


Load in Pounds
16000
14000
12000
10000
8000
6000
4000
20 00
Elongation in Inches
0.10
0.50
0.20
0.10
0.30
0.15
0.40
0.20
0.50
0.25
0.60
0.30
TEST NO 11
1/2 in Rope 12 in Sheave

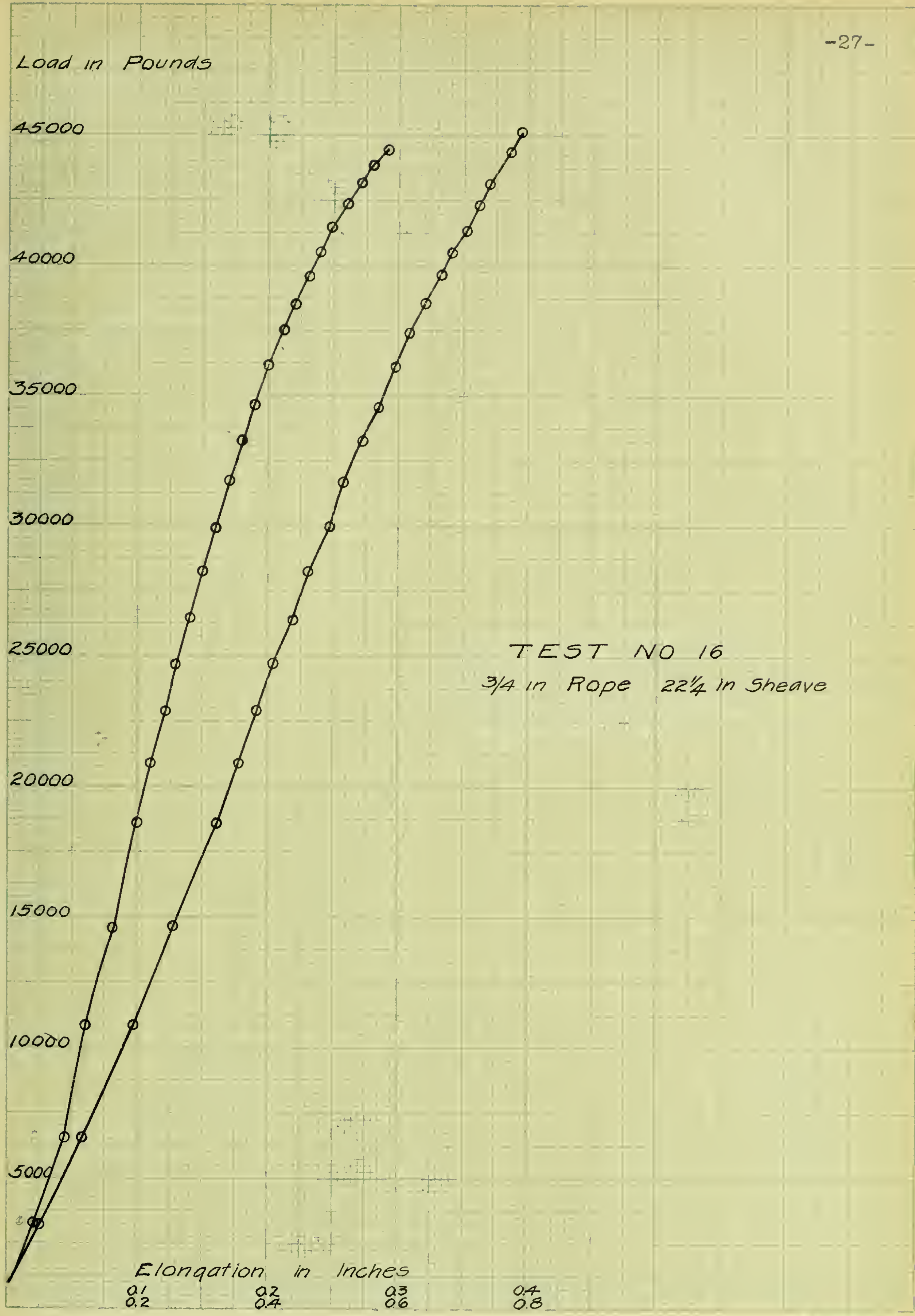

Load in Pounds
45000
40000
35000
30000
25000
20000
15000
10000
5000
TEST NO 12
3/4 in Rope 18 in Sheave
Elongation in Inches
0.1
0.2
0.3
0.4
0.5
0.6

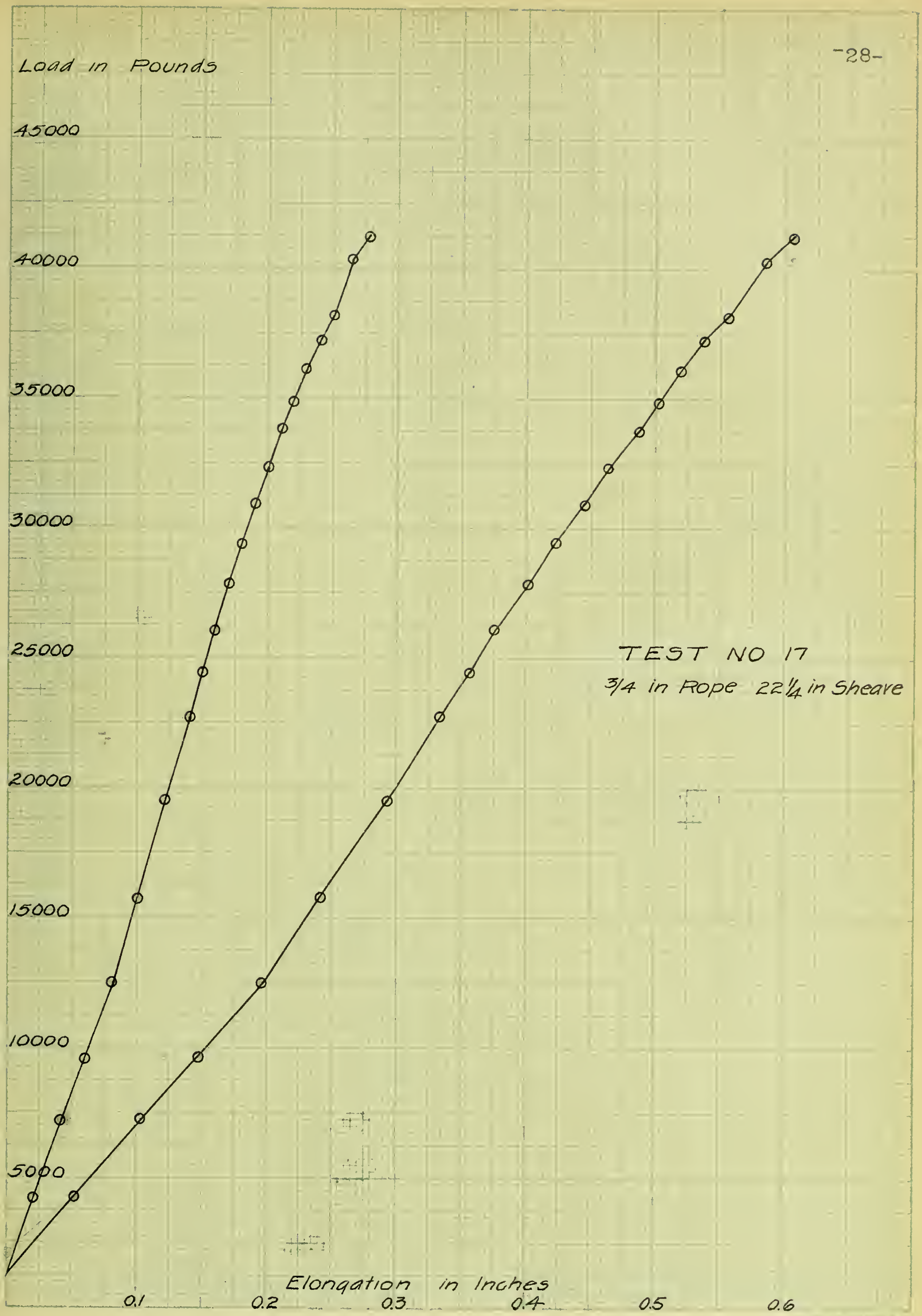

Load in Pounds
45000
40000
35000
30000
25000
20000
15000
10000
5000
TEST NO 13
3/4 in Rope 18 in Sheave
Elongation in Inches
0.1
0.2
0.3
0.4
0.5
0.6

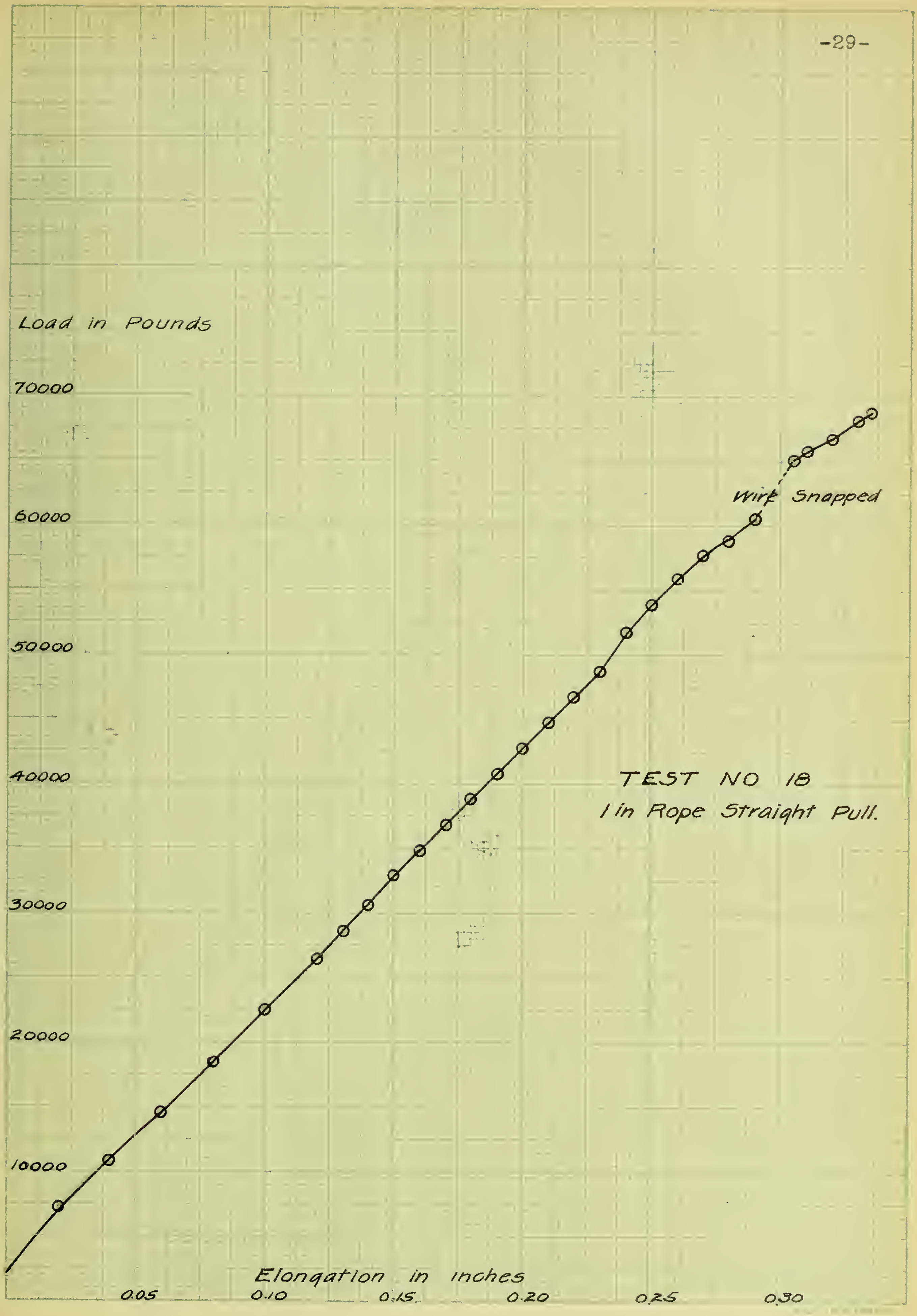

Load in Pounds
45000
40000
35000
30000
25000
20000
15000
10000
5000
Elongation in Inches
0.1
0.2
0.3
0.4
0.5
TEST NO 14
3/4 in Rope 18 in Sheave

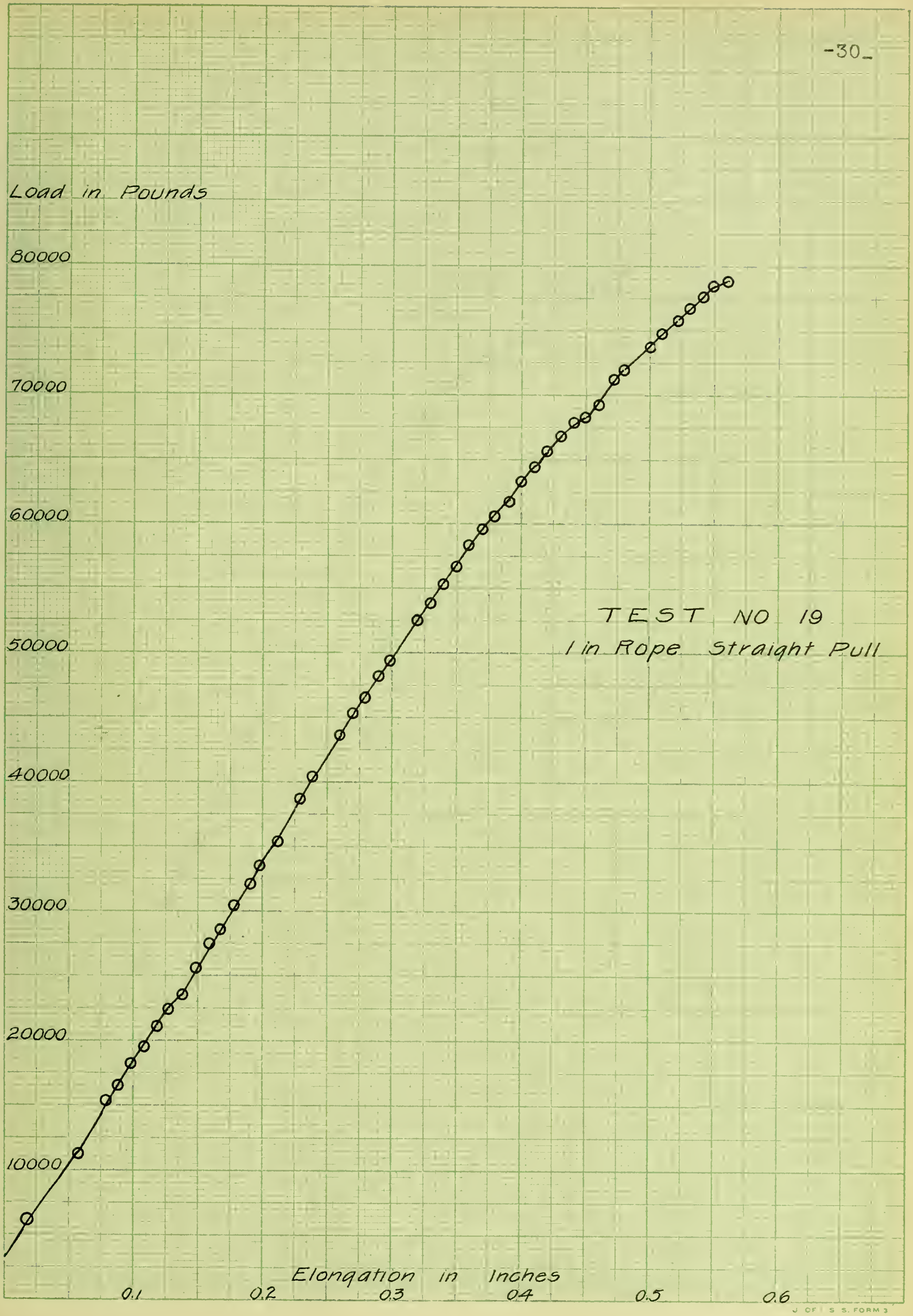


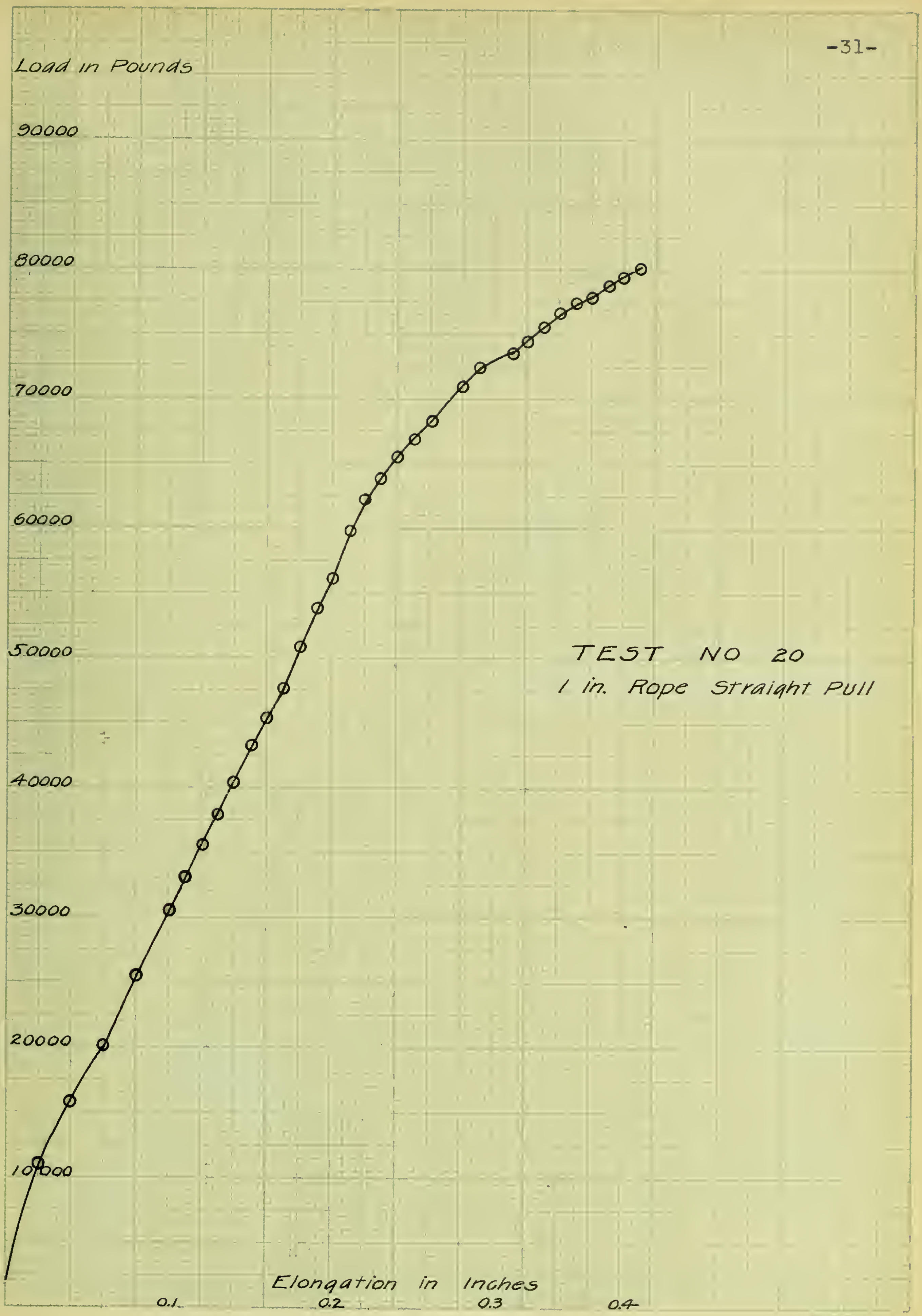
Load in Pounds
45000
40000
35000
30000
25000
20000
15000
10000
5000
TEST NO 15
3/4 in Rope 22¼ in Sheave
Elongation in Inches
0.1
0.2
0.3
0.4
0.5

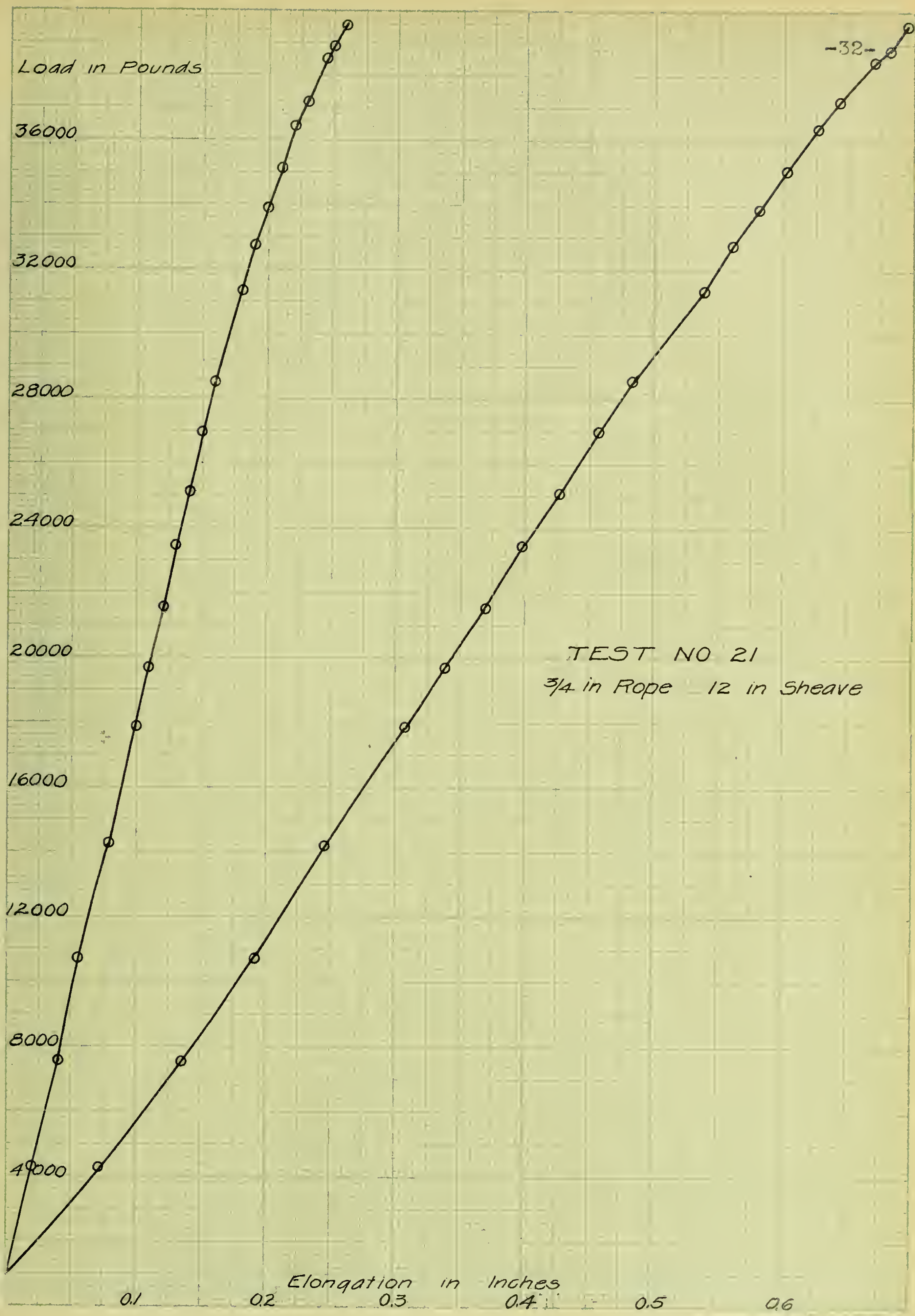

Load in Pounds
45000
40000
35000
30000
25000
20000
15000
10000
5000
TEST NO 16
3/4 in Rope 22¼ in Sheave
Elongation in Inches
0.1
0.2
0.2
0.4
0.3
0.6
0.4
0.8

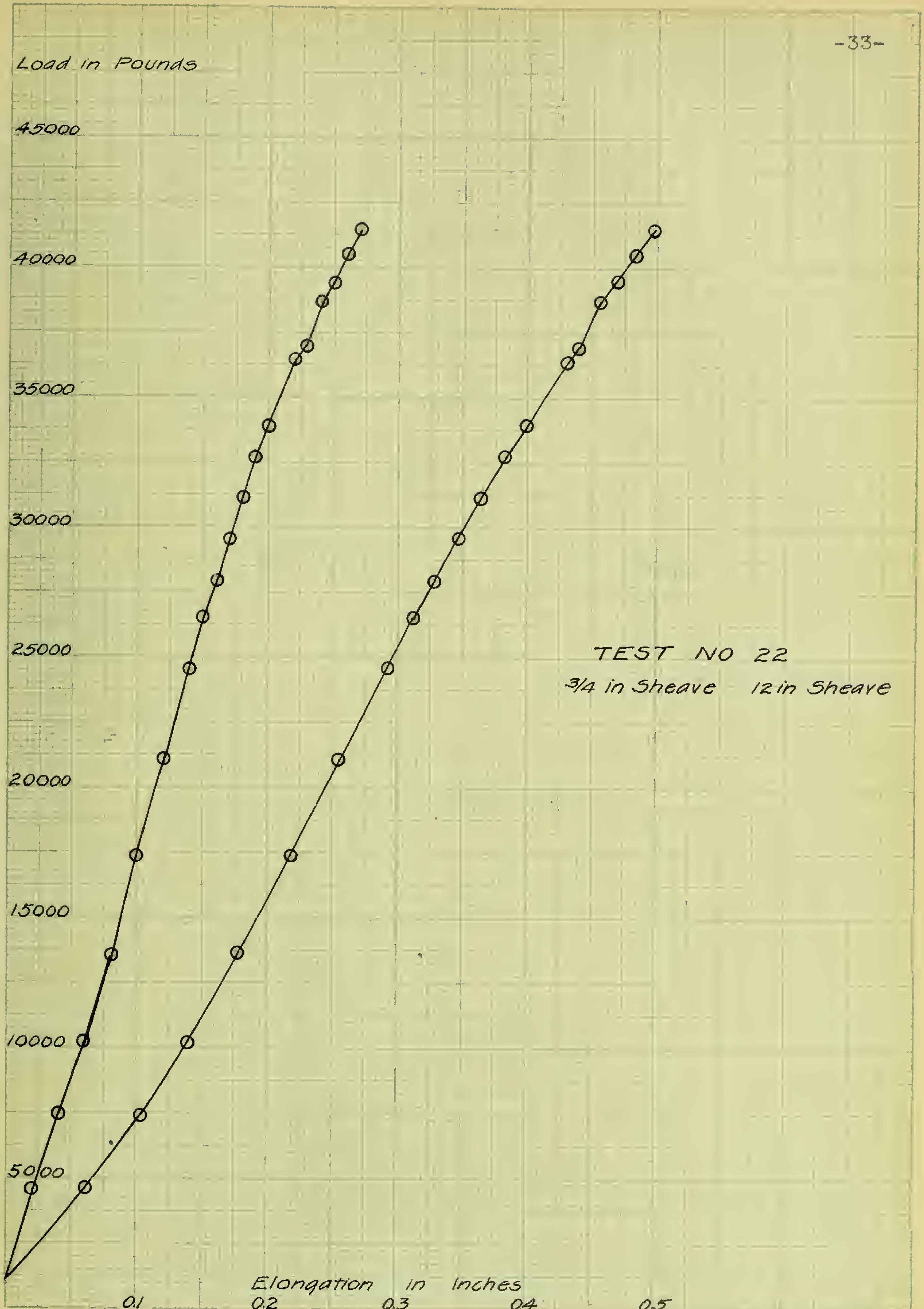

Load in Pounds
45000
40000
35000
30000
25000
20000
15000
10000
5000
TEST NO 17
3/4 in Rope 22 1/4 in Sheave
Elongation in Inches
0.1
0.2
0.3
0.4
0.5
0.6

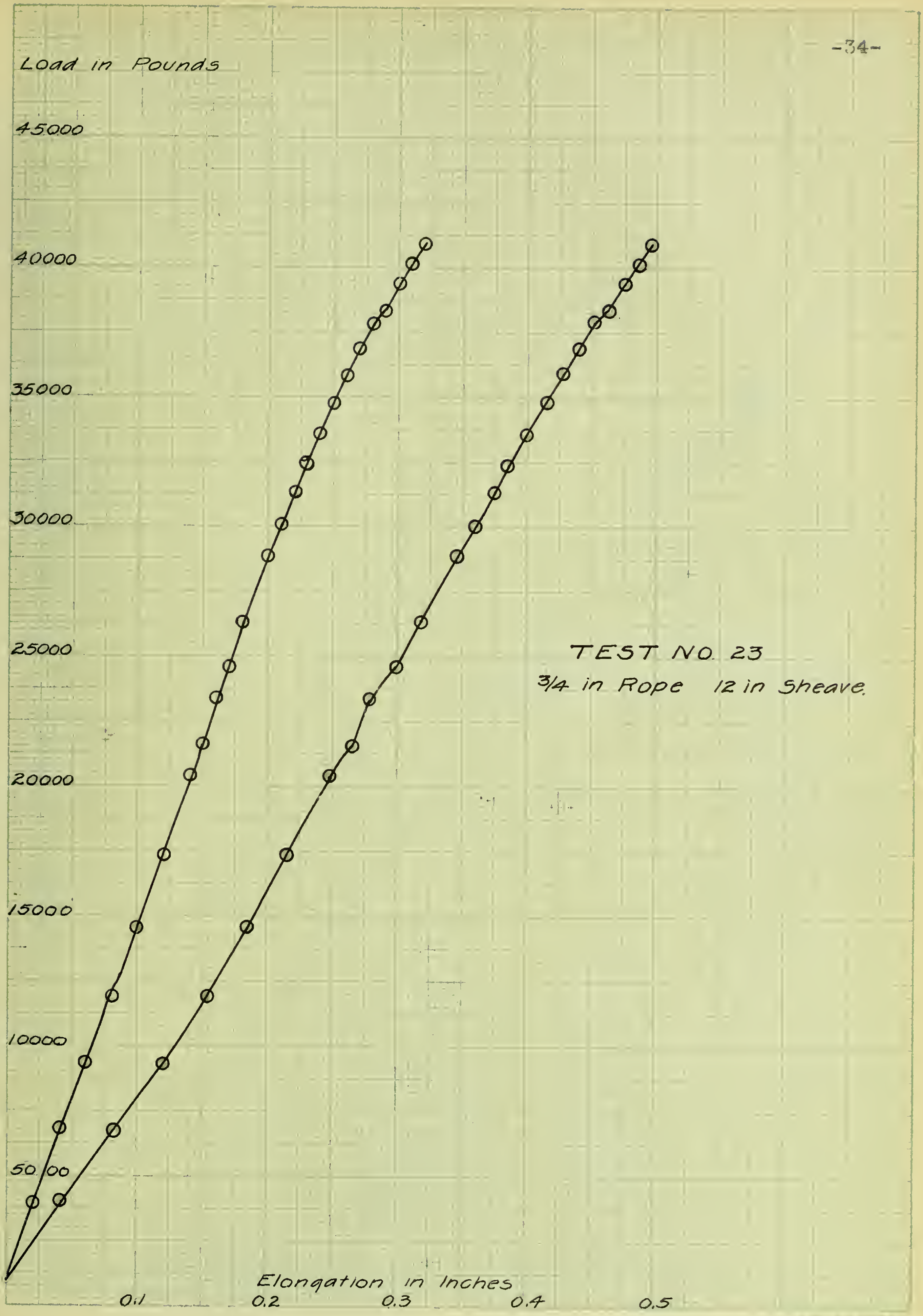

Load in Pounds
70000
60000
50000
40000
30000
20000
10000
Wire Snapped
TEST NO 18
1 in Rope Straight Pull.
Elongation in inches
0.05
0.10
0.15
0.20
0.25
0.30


Load in Pounds
80000
70000
60000
50000
40000
30000
20000
10000
TEST NO 19
1 in Rope Straight Pull
Elongation in Inches
0.1
0.2
0.3
0.4
0.5
0.6

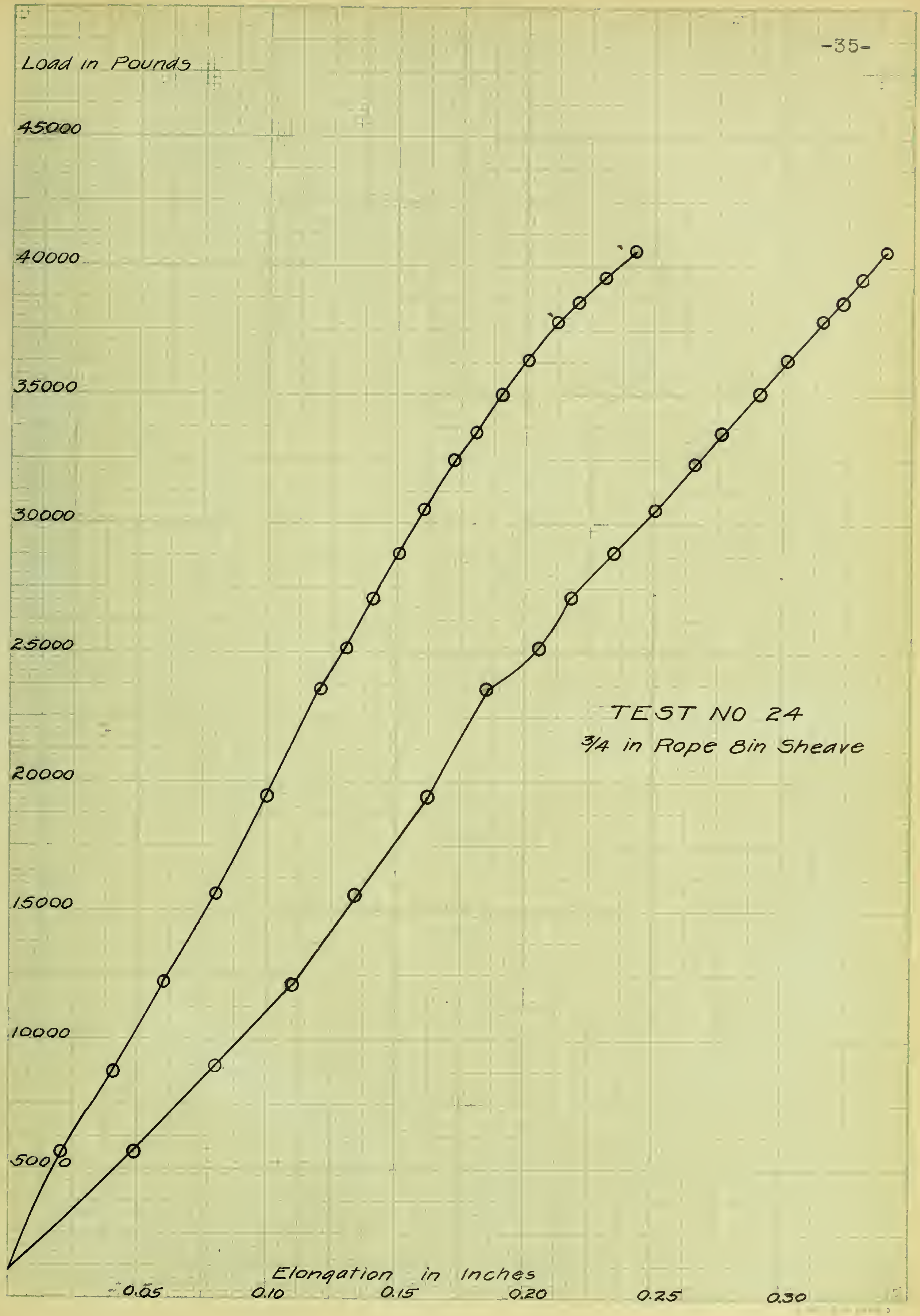

Load in Pounds
90000
80000
70000
60000
50000
40000
30000
20000
10000
TEST NO 20
1 in. Rope Straight Pull
Elongation in Inches
0.1
0.2
0.3
0.4

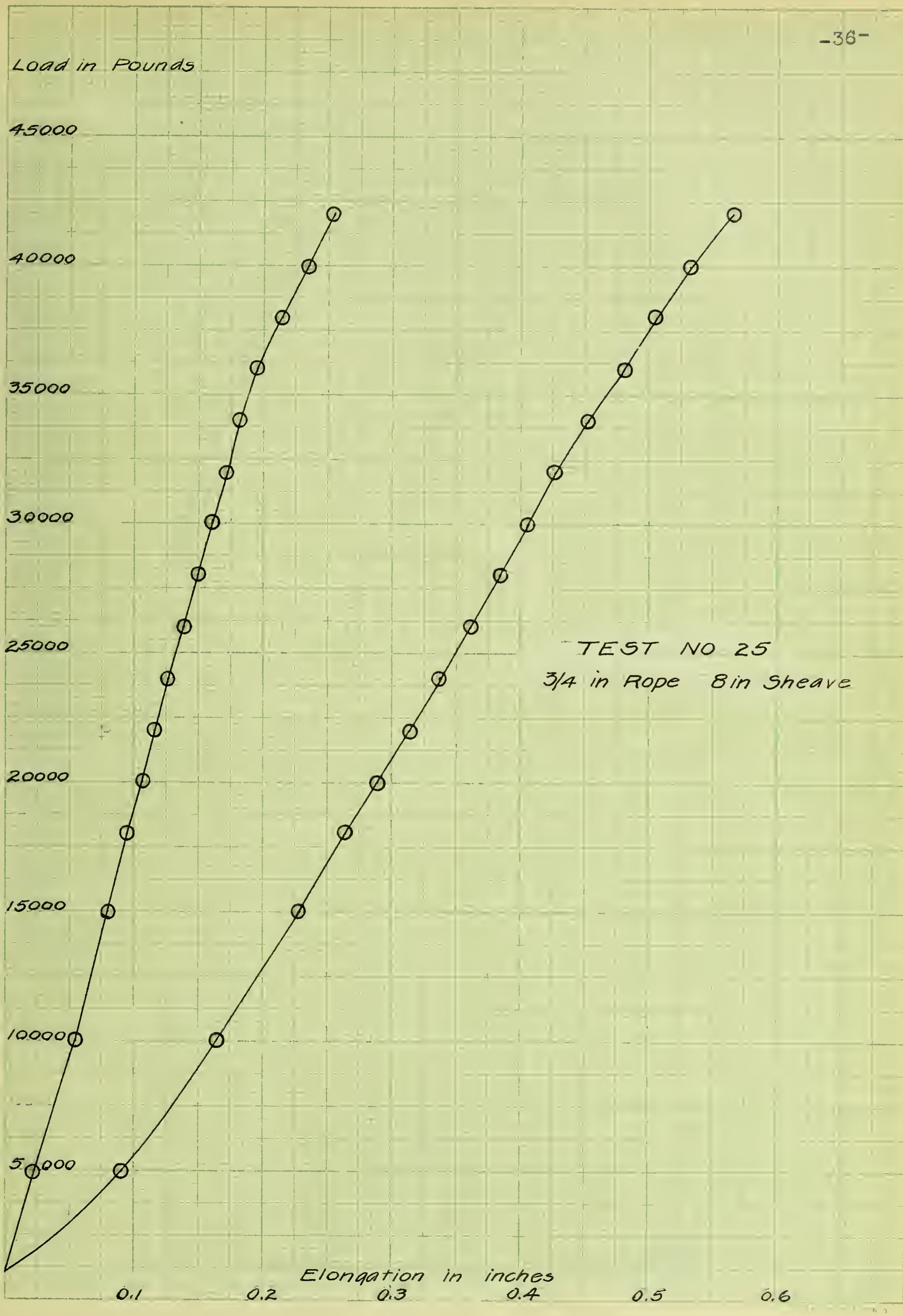

Load in Pounds
36000
32000
28000
24000
20000
16000
12000
8000
4000
Elongation in Inches
0.1
0.2
0.3
0.4
0.5
0.6
TEST NO 21
3/4 in Rope 12 in Sheave

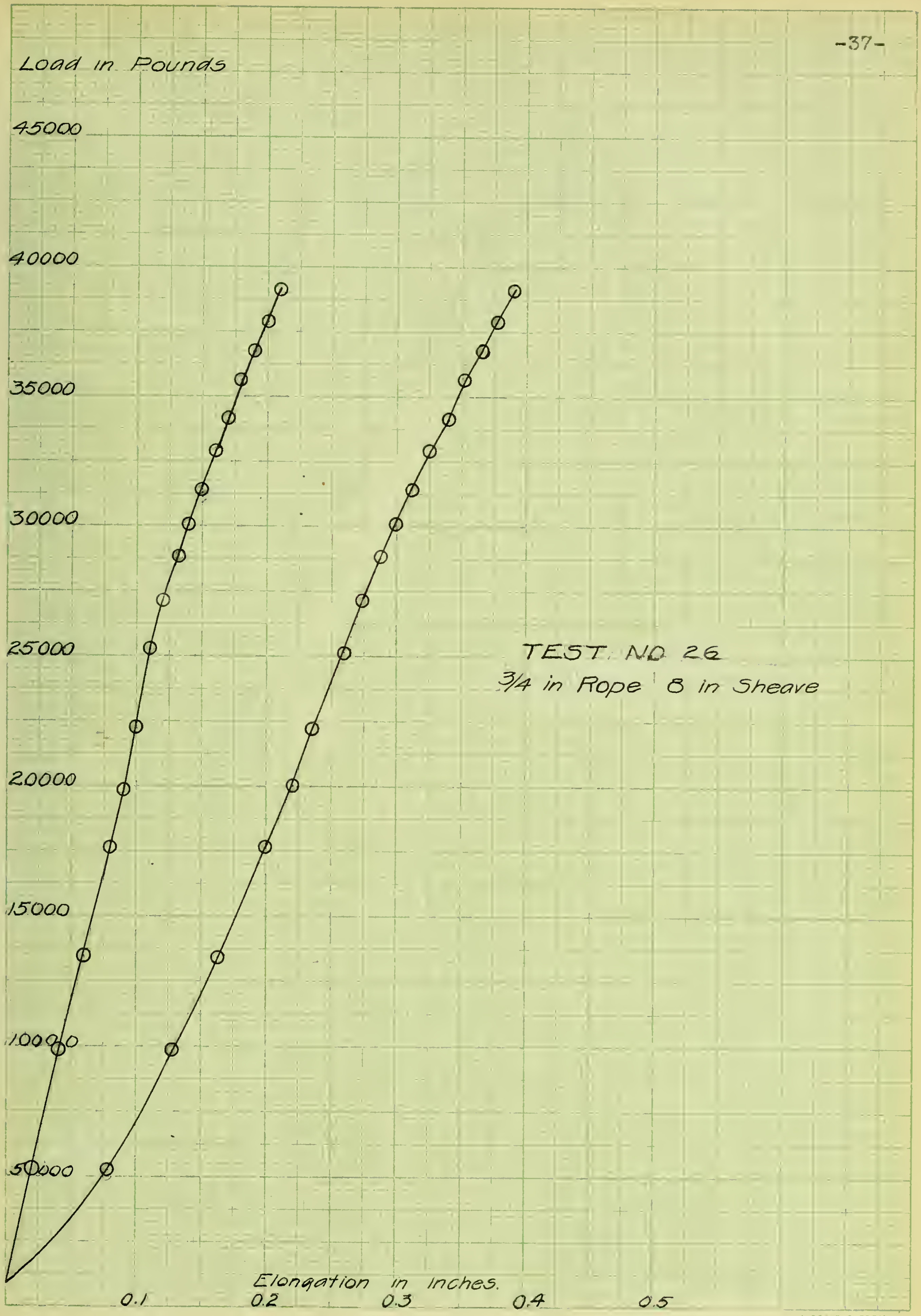

Load in Pounds
45000
40000
35000
30000
25000
20000
15000
10000
5000
TEST NO 22
3/4 in Sheave 12 in Sheave
Elongation in Inches
0.1
0.2
0.3
0.4
0.5

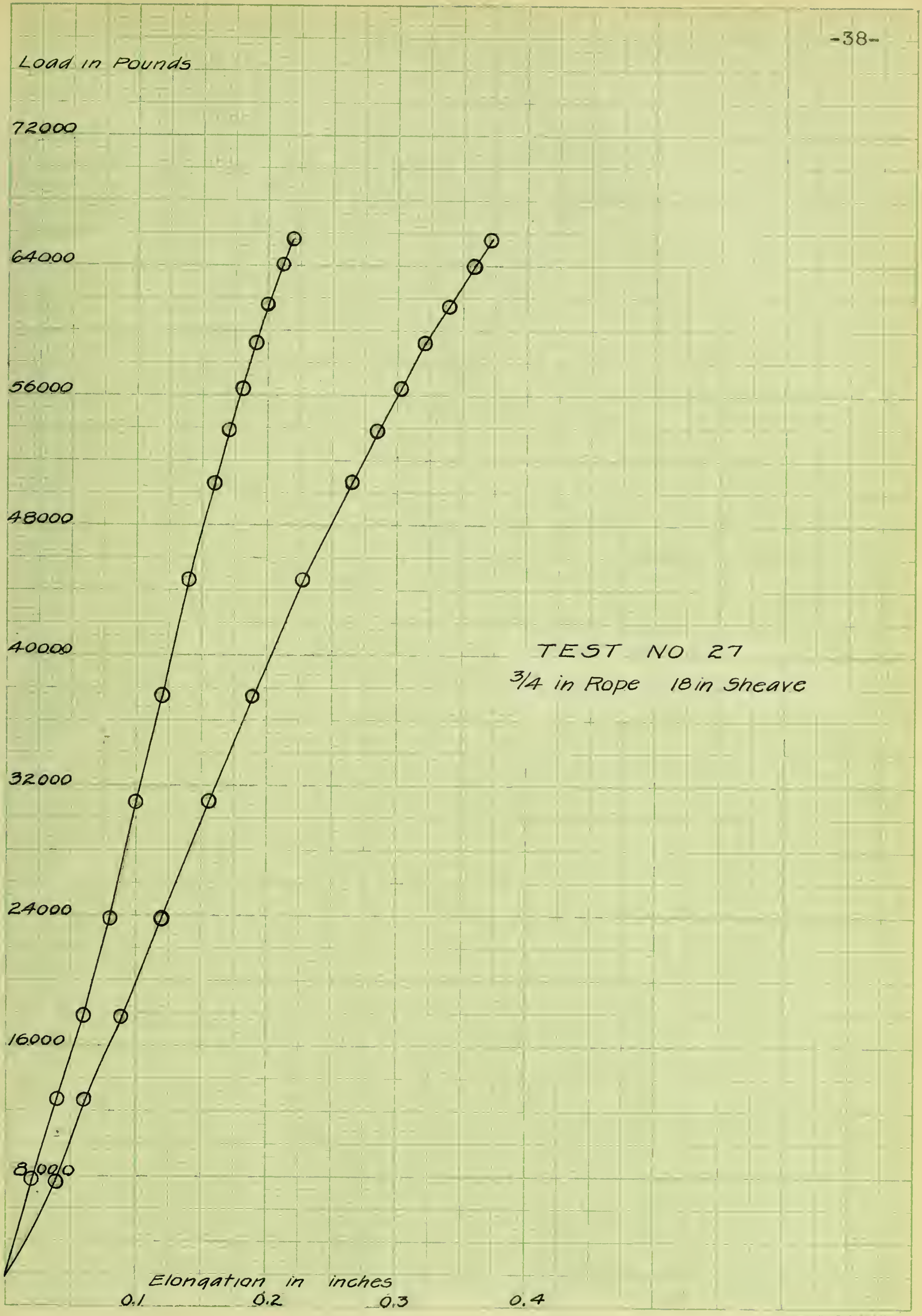


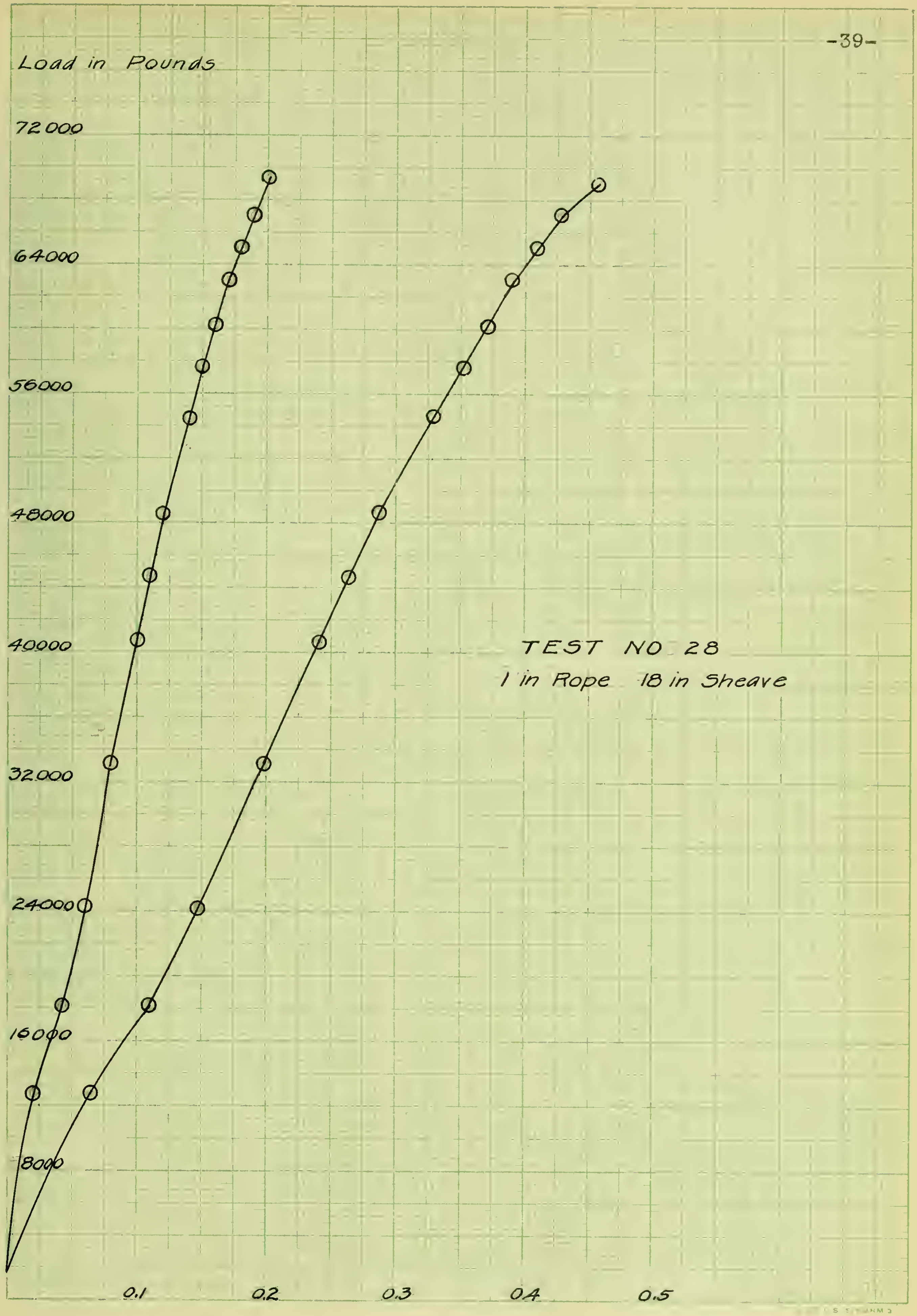
Load in Pounds
45000
40000
35000
30000
25000
20000
15000
10000
50.00
TEST NO. 23
3/4 in Rope 12 in Sheave.
Elongation in Inches
0.1
0.2
0.3
0.4
0.5

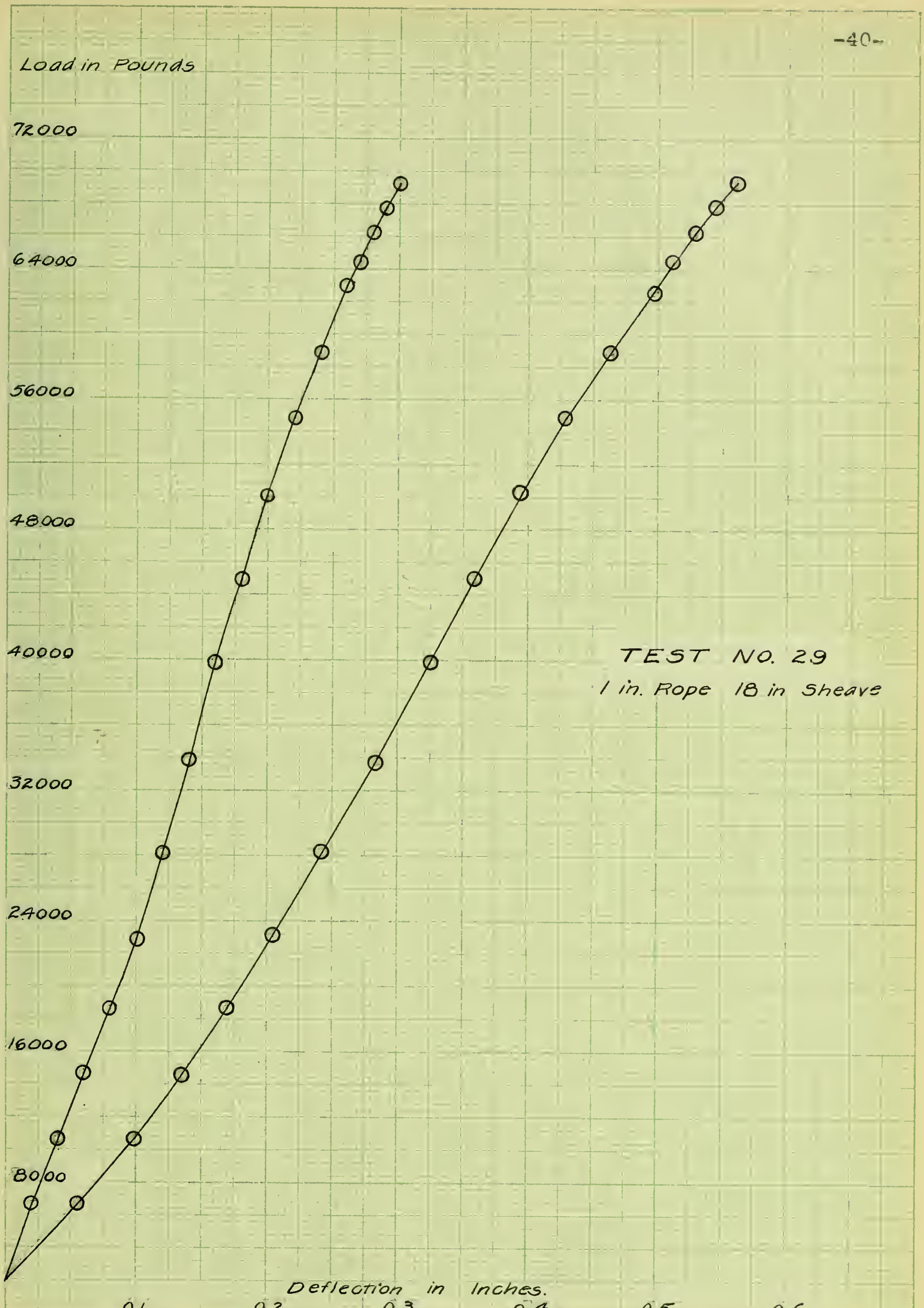

Load in Pounds
45000
40000
35000
30000
25000
20000
15000
10000
5000
TEST NO 24
3/4 in Rope 8in Sheave
Elongation in Inches
0.05
0.10
0.15
0.20
0.25
0.30

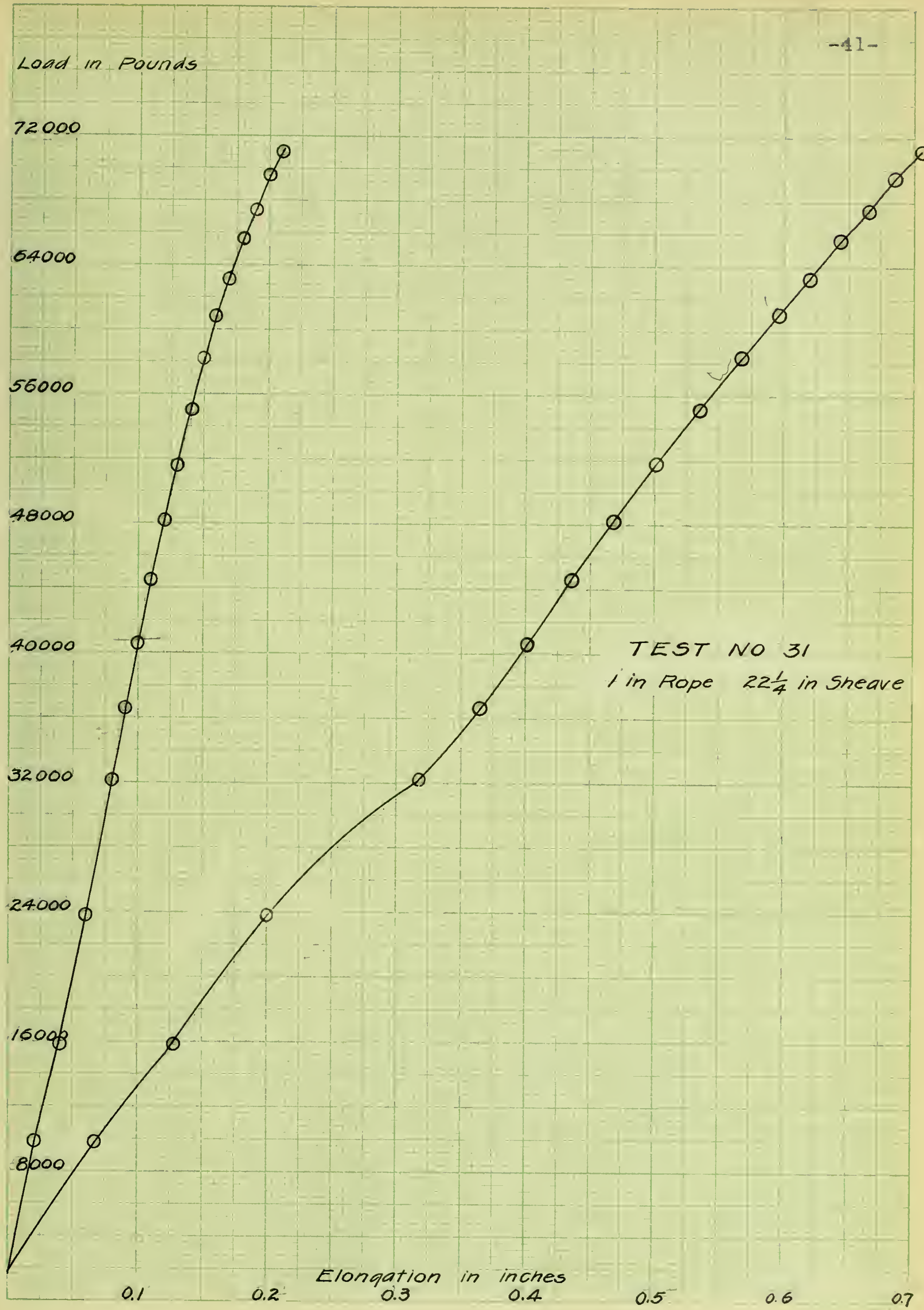

Load in Pounds
45000
40000
35000
30000
25000
20000
15000
10000
5000
TEST NO 25
3/4 in Rope 8 in Sheave
Elongation in inches
0.1
0.2
0.3
0.4
0.5
0.6

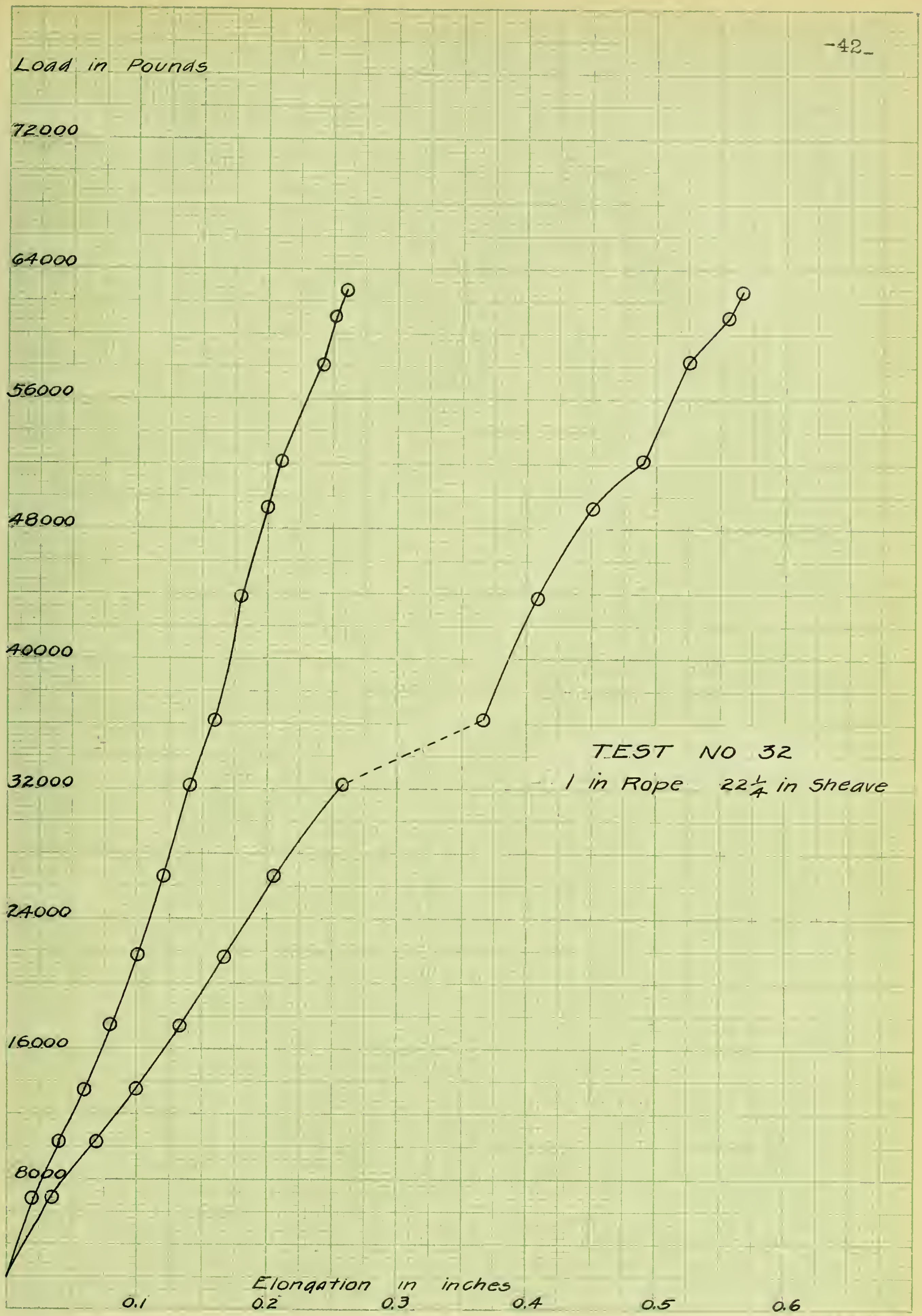

Load in Pounds
45000
40000
35000
30000
25000
20000
15000
10000
5000
TEST NO 26
3/4 in Rope 8 in Sheave
Elongation in inches.
0.1
0.2
0.3
0.4
0.5

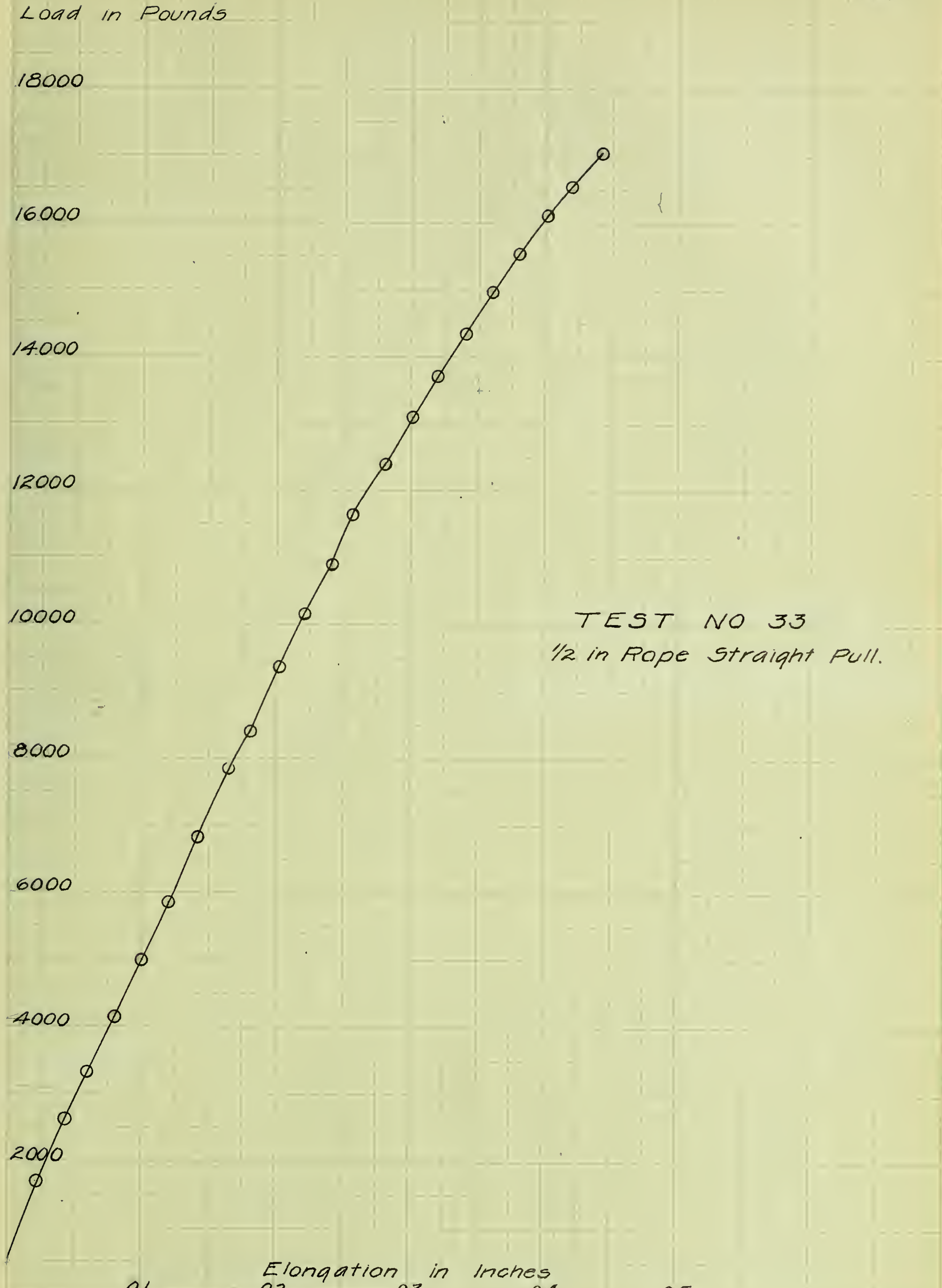

Load in Pounds
72000
64000
56000
48000
40000
32000
24000
16000
8000
TEST NO 27
3/4 in Rope 18 in Sheave
Elongation in inches
0.1
0.2
0.3
0.4

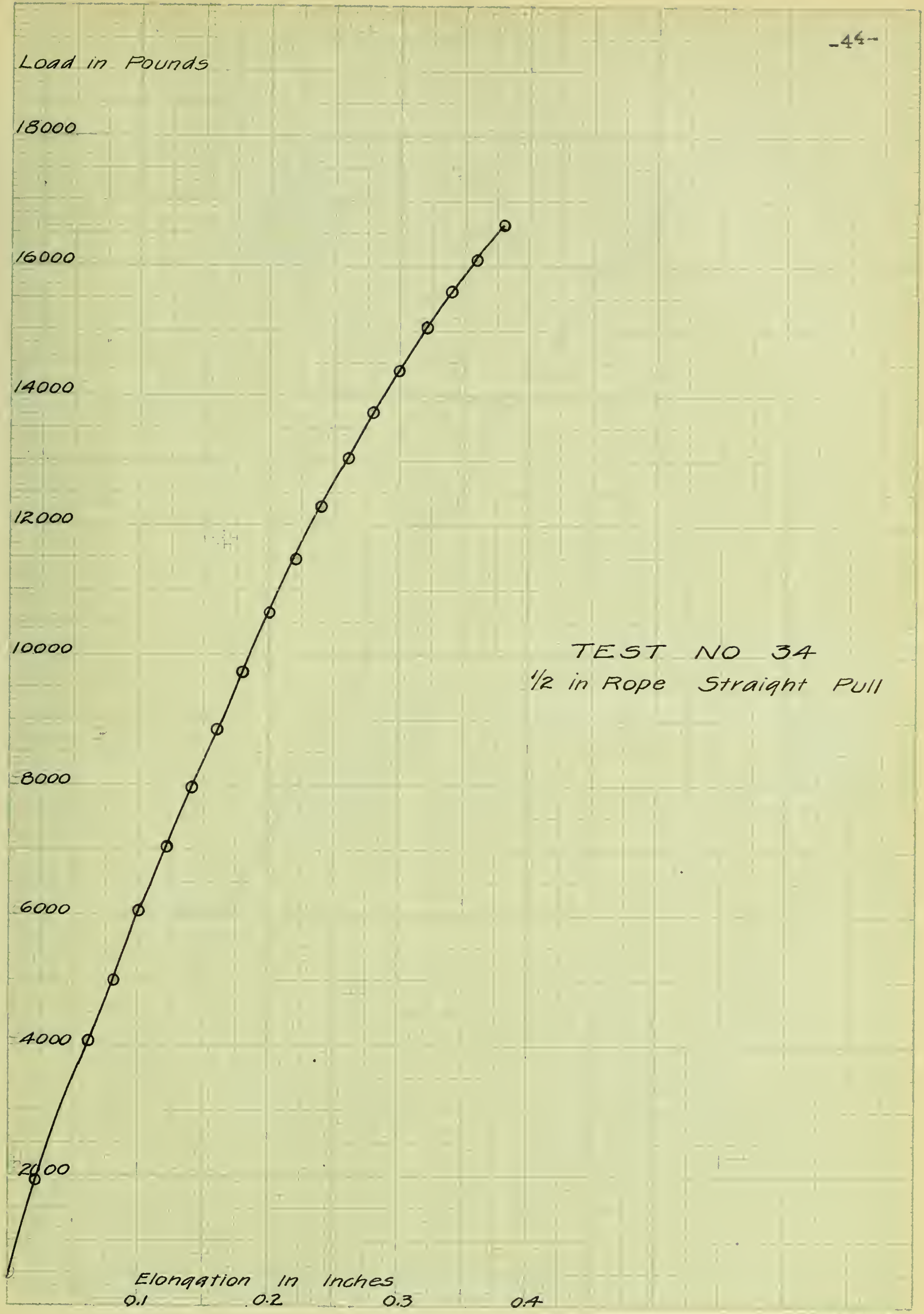

Load in Pounds
72000
64000
56000
48000
40000
32000
24000
16000
8000
0.1
0.2
0.3
0.4
0.5
TEST NO 28
1 in Rope 18 in Sheave

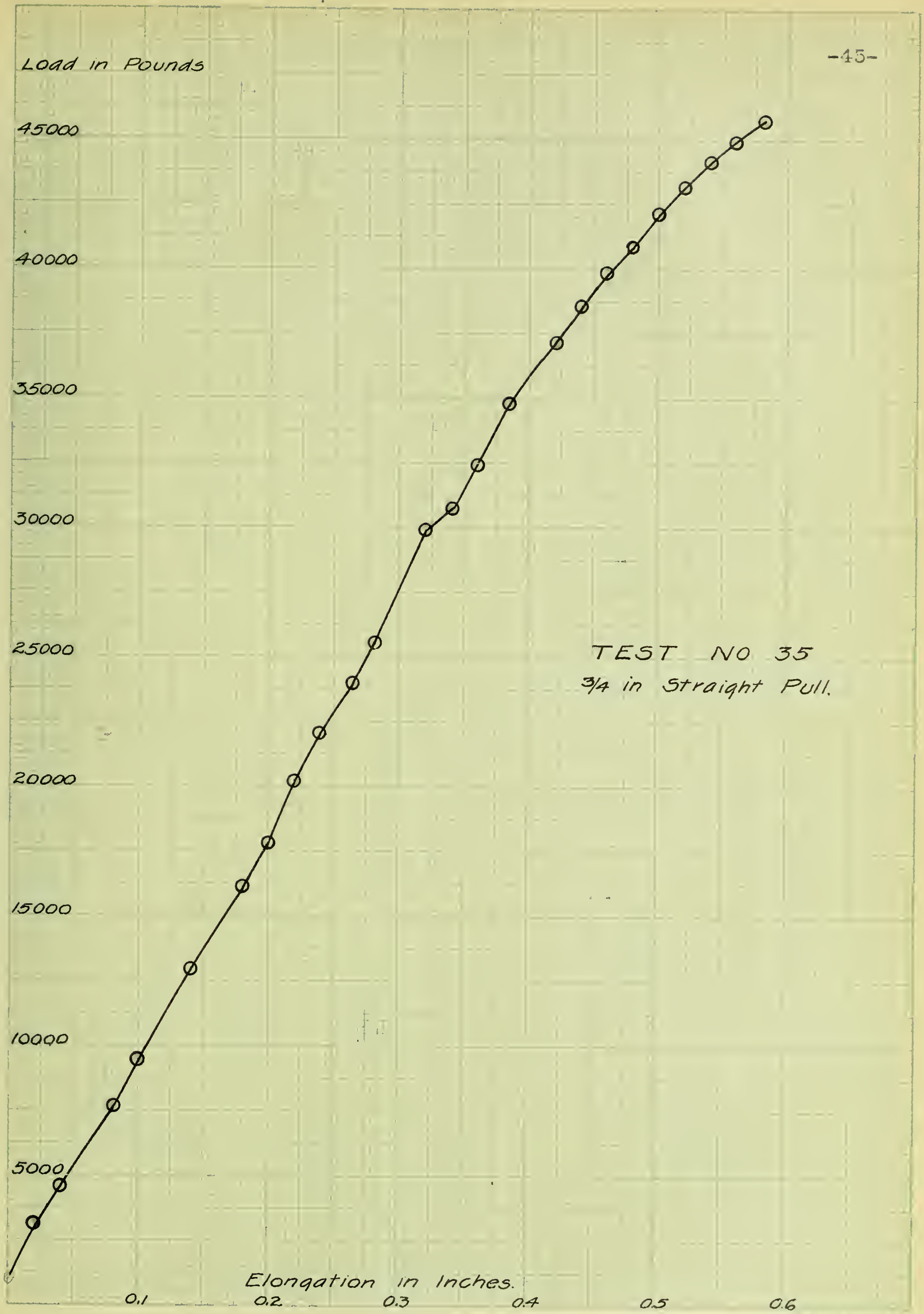

Load in Pounds
72000
64000
56000
48000
40000
32000
24000
16000
8000
TEST NO. 29
1 in. Rope 18 in Sheave
Deflection in Inches.
0.1
0.2
0.3
0.4
0.5
0.6

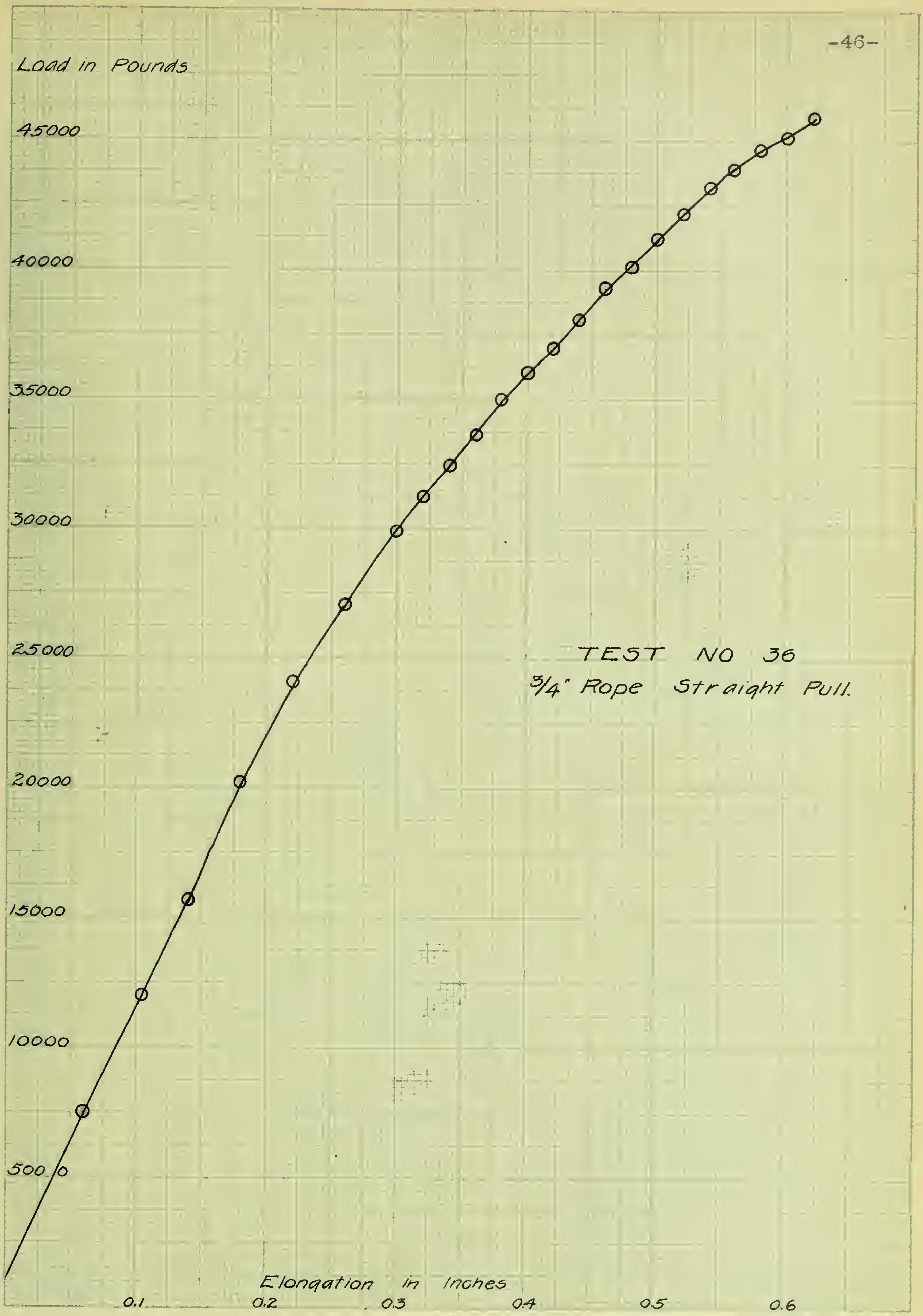

Load in Pounds
72000
64000
56000
48000
40000
32000
24000
16000
8000
TEST NO 31
1 in Rope 22 1/4 in Sheave
Elongation in inches
0.1
0.2
0.3
0.4
0.5
0.6
0.7

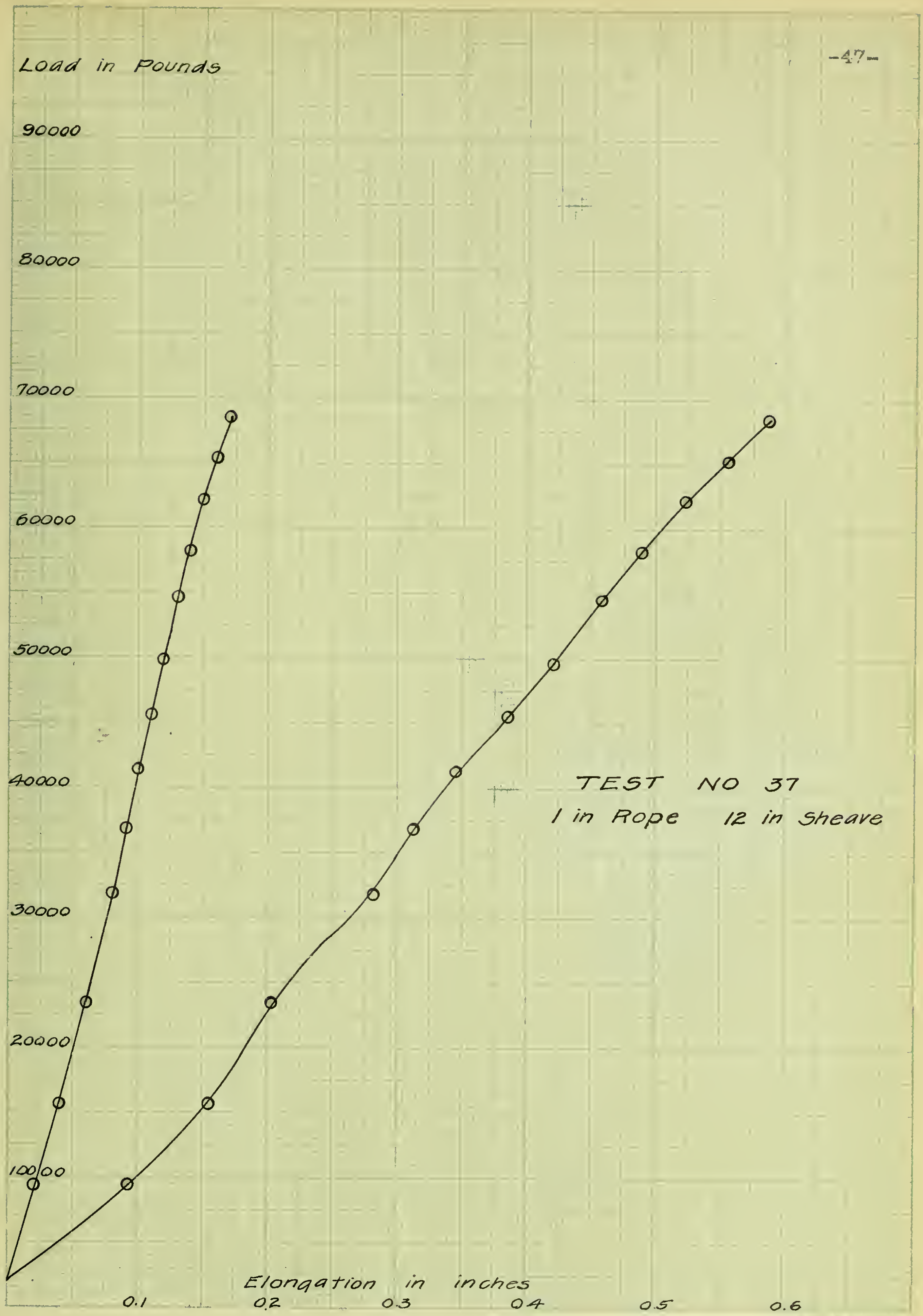

Load in Pounds
72000
64000
56000
48000
40000
32000
24000
16000
8000
TEST NO 32
1 in Rope 22 1/4 in Sheave
Elongation in inches
0.1
0.2
0.3
0.4
0.5
0.6

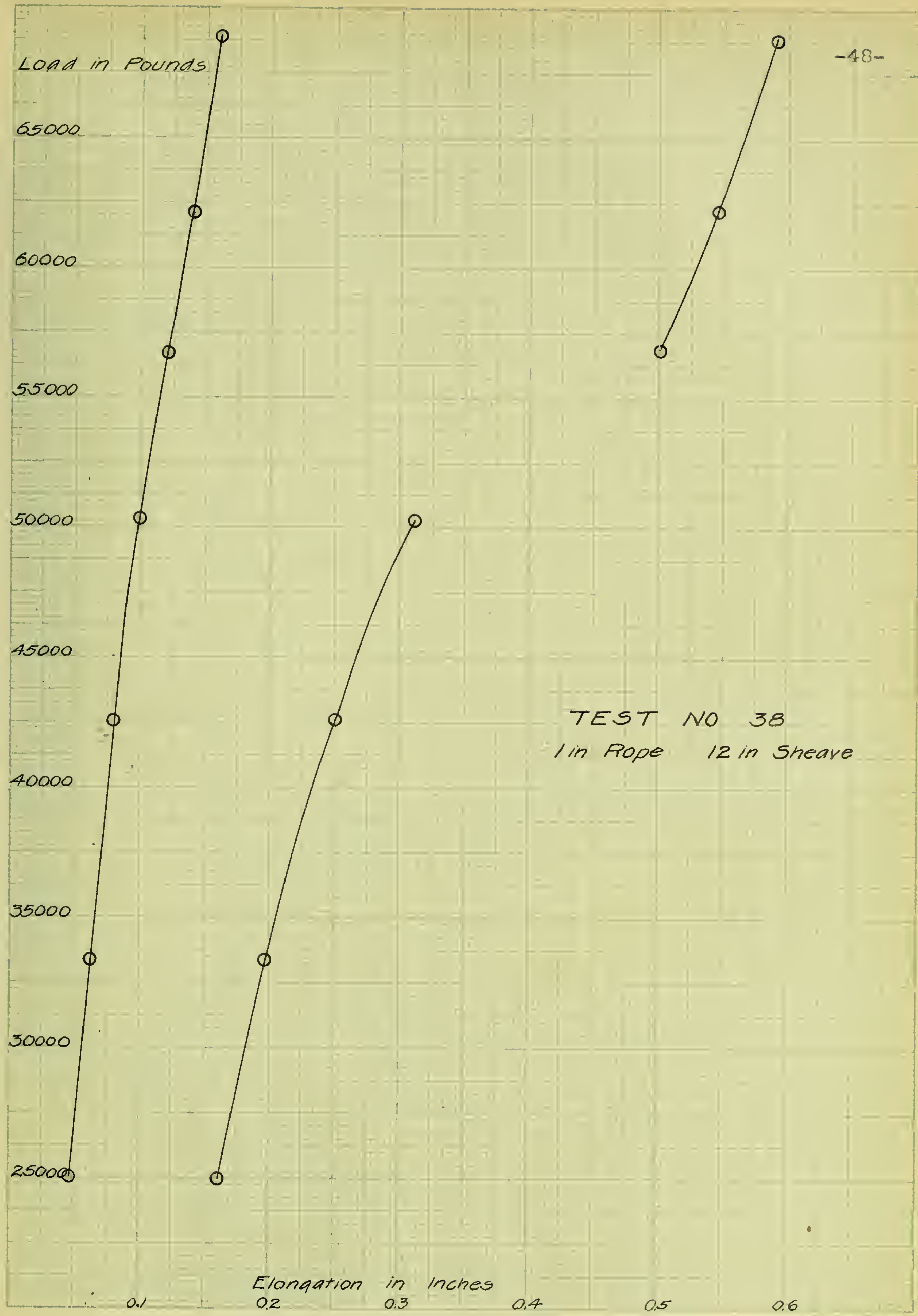

Load in Pounds
18000
16000
14000
12000
10000
8000
6000
4000
2000
TEST NO 33
1/2 in Rope Straight Pull.
Elongation in Inches
0.1
0.2
0.3
0.4
0.5

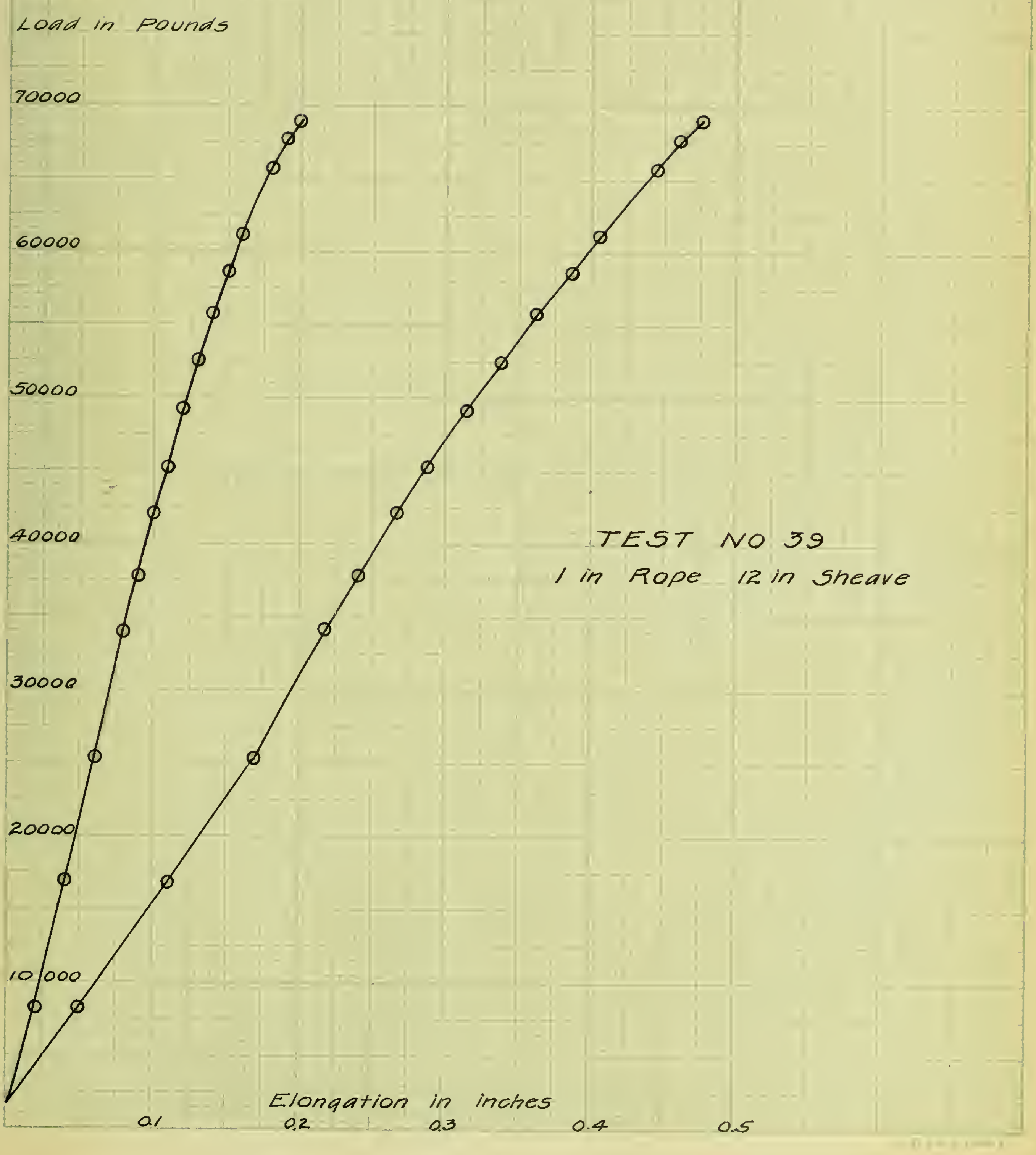

Load in Pounds
18000
16000
14000
12000
10000
8000
6000
4000
2000
Elongation in Inches
0.1
0.2
0.3
0.4
TEST NO 34
1/2 in Rope Straight Pull


Load in Pounds
45000
40000
35000
30000
25000
20000
15000
10000
5000
TEST NO 35
3/4 in Straight Pull.
Elongation in Inches.
0.1
0.2
0.3
0.4
0.5
0.6


Load in Pounds
45000
40000
35000
30000
25000
20000
15000
10000
500 0
TEST NO 36
3/4" Rope Straight Pull.
Elongation in Inches
0.1
0.2
0.3
0.4
0.5
0.6


Load in Pounds
90000
80000
70000
60000
50000
40000
30000
20000
10000
TEST NO 37
1 in Rope 12 in Sheave
Elongation in inches
0.1
0.2
0.3
0.4
0.5
0.6

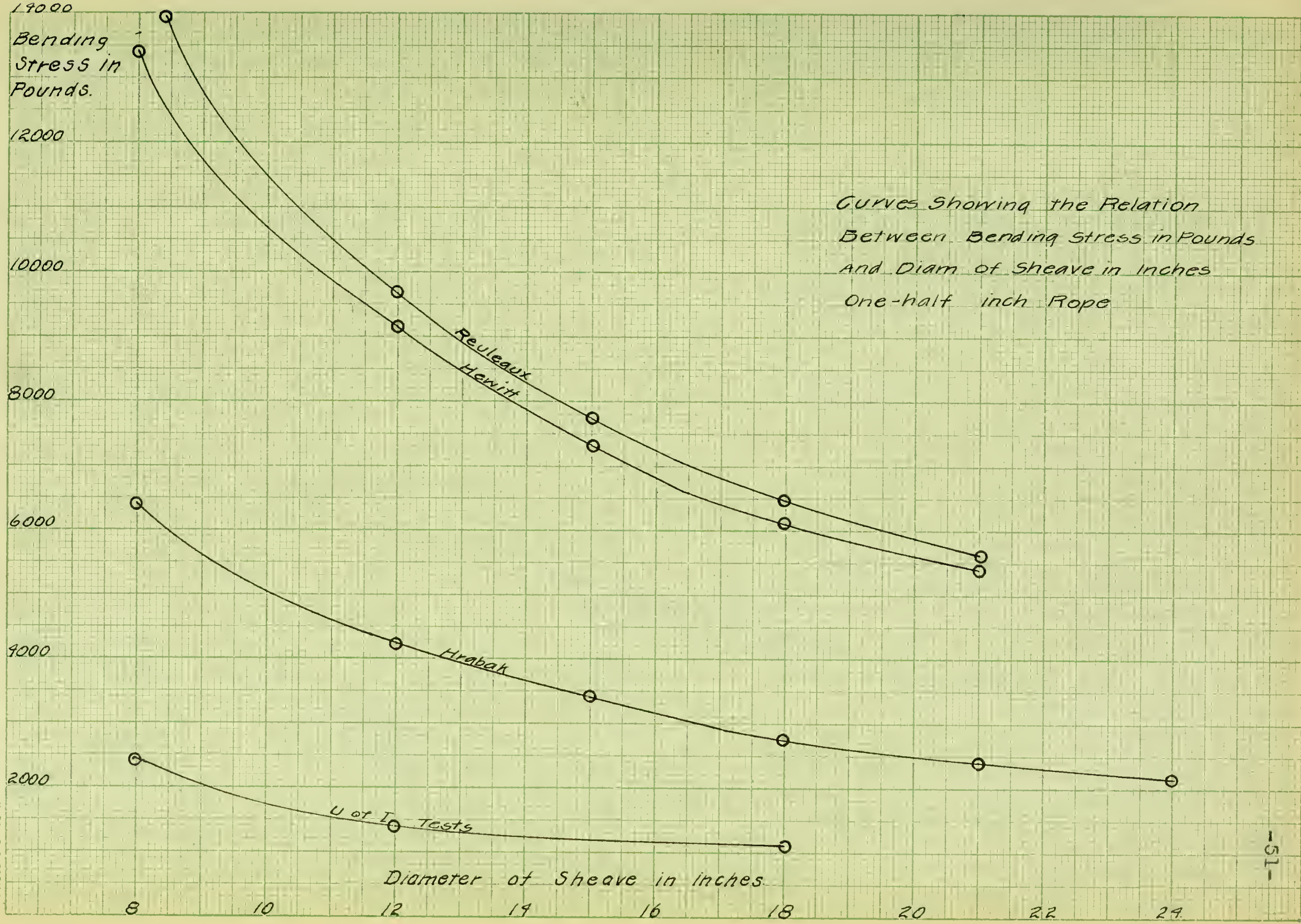

Load in Pounds
65000
60000
55000
50000
45000
40000
35000
30000
25000
TEST NO 38
1 in Rope 12 in Sheave
Elongation in Inches
0.1
0.2
0.3
0.4
0.5
0.6

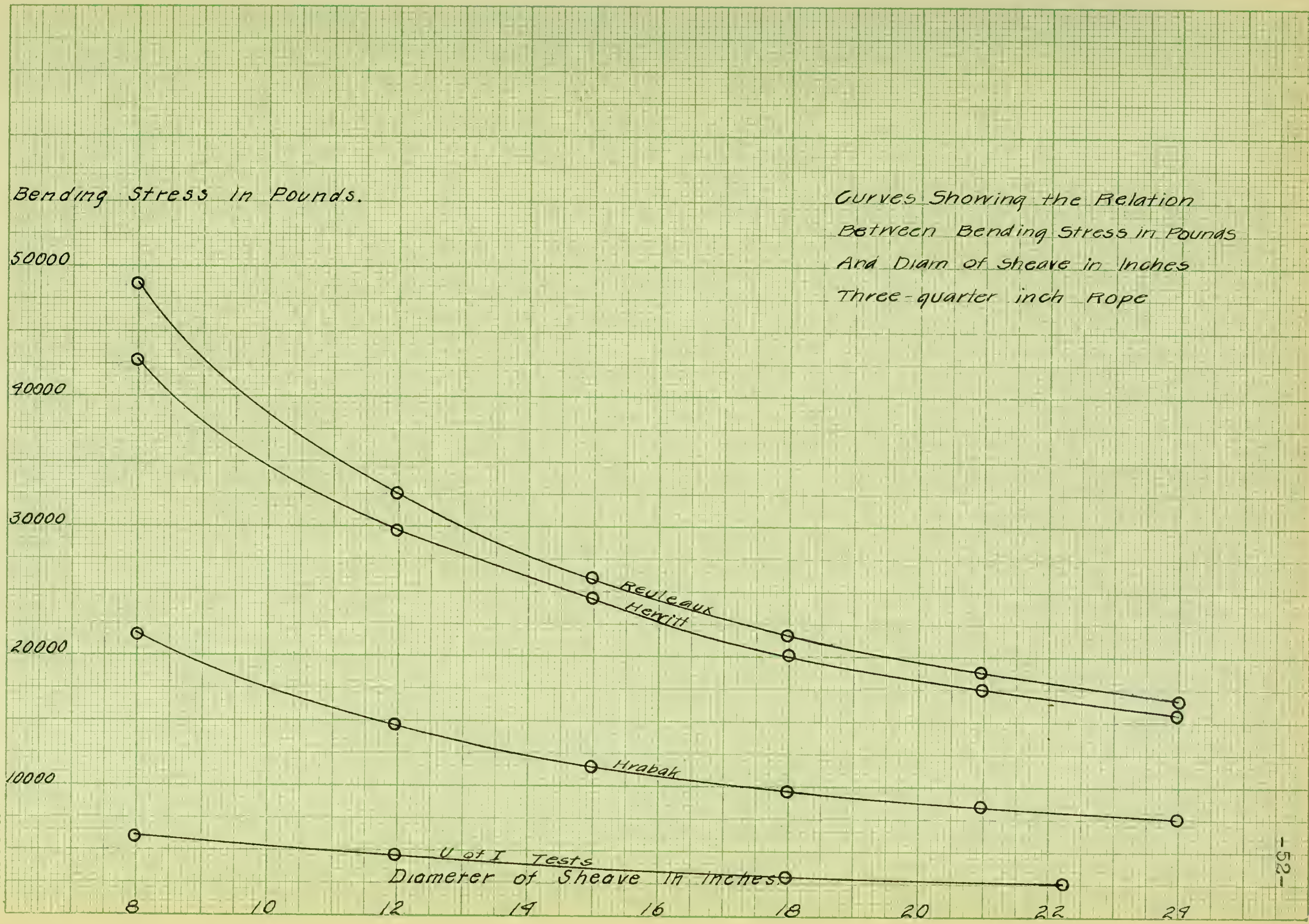

Load in Pounds
70000
60000
50000
40000
30000
20000
10 000
Elongation in inches
0.1
0.2
0.3
0.4
0.5
TEST NO 39
1 in Rope 12 in Sheave

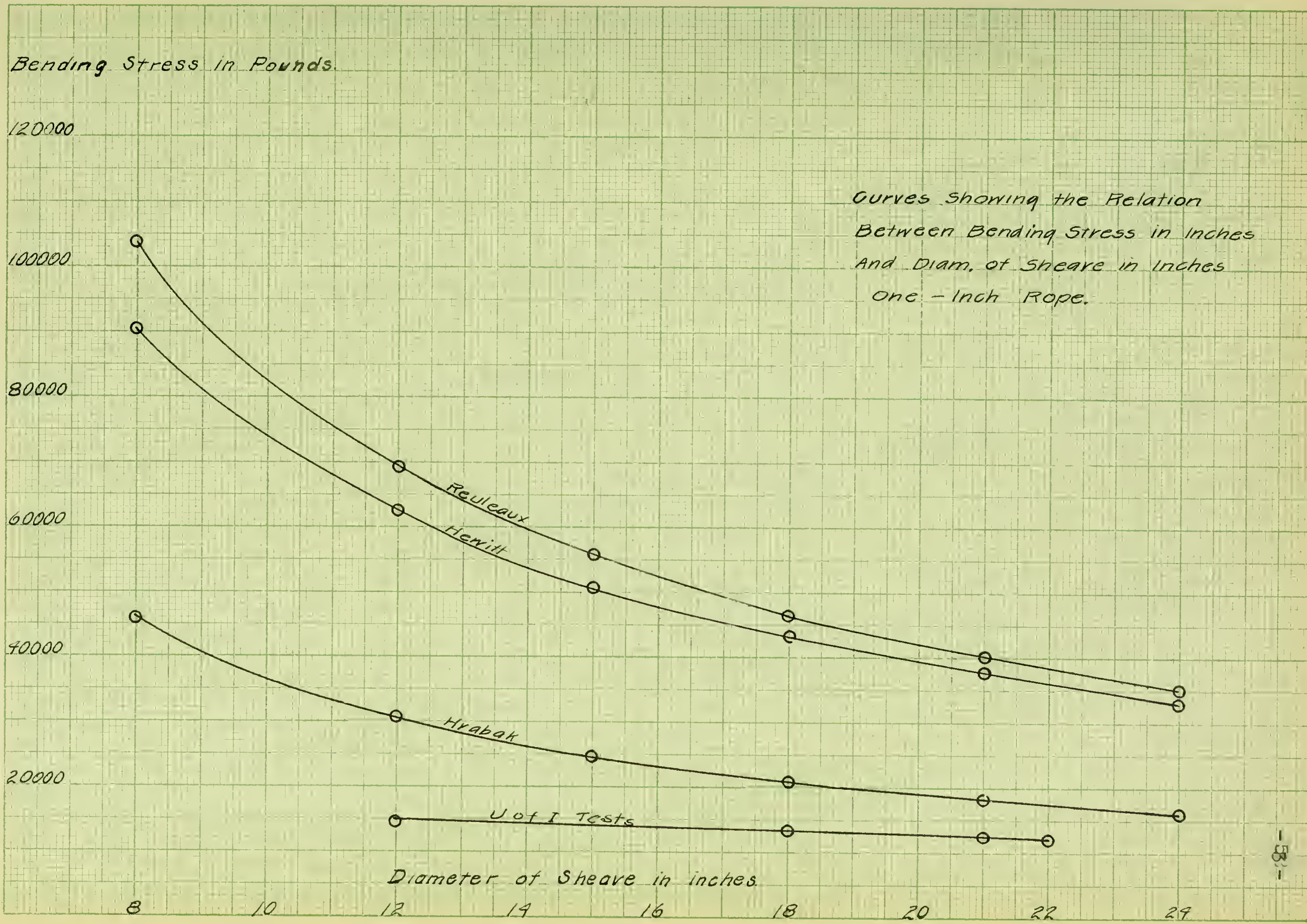
CURVES SHOWING THE RELATION

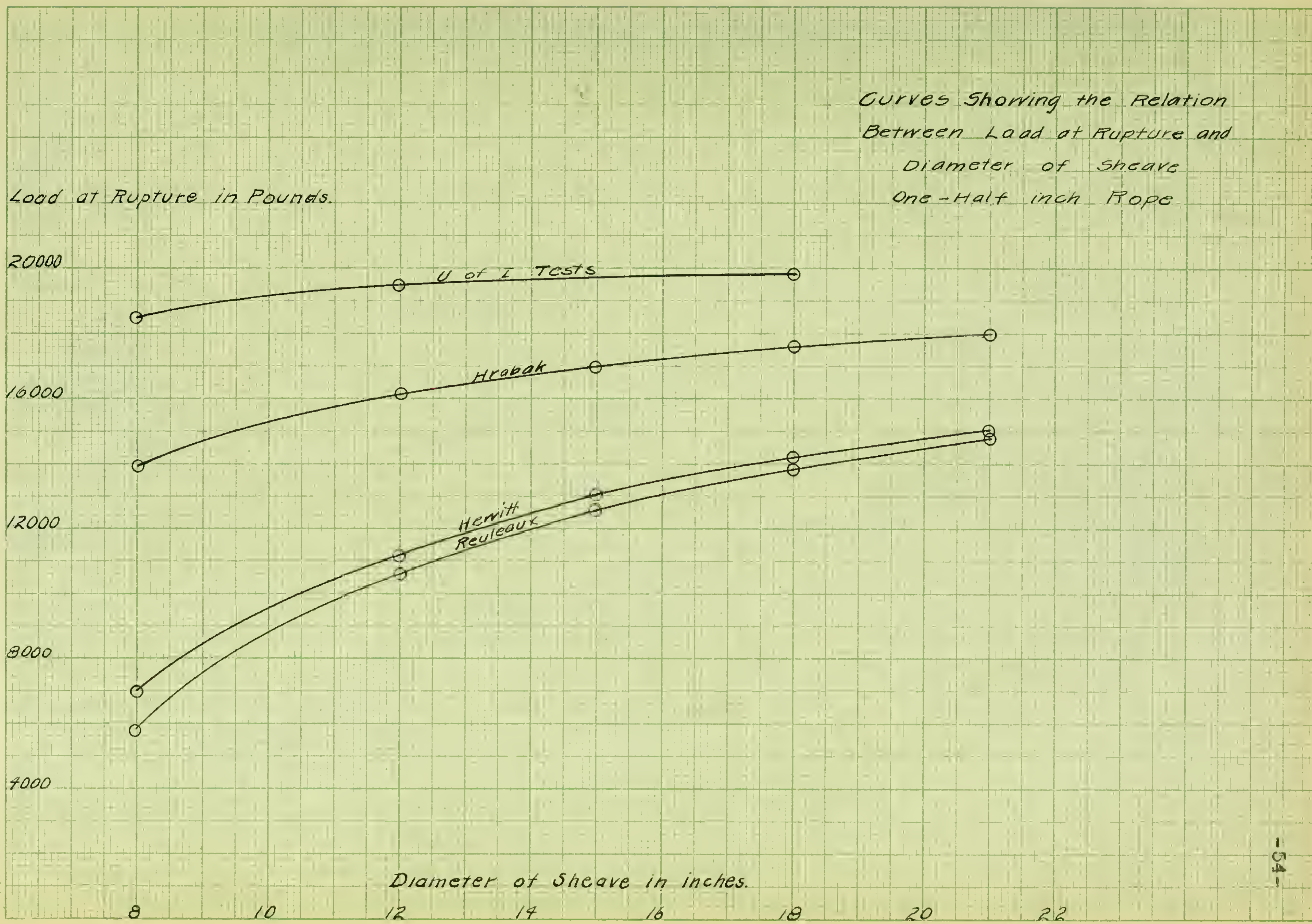
BETWEEN TEST VALUES AND THOSE

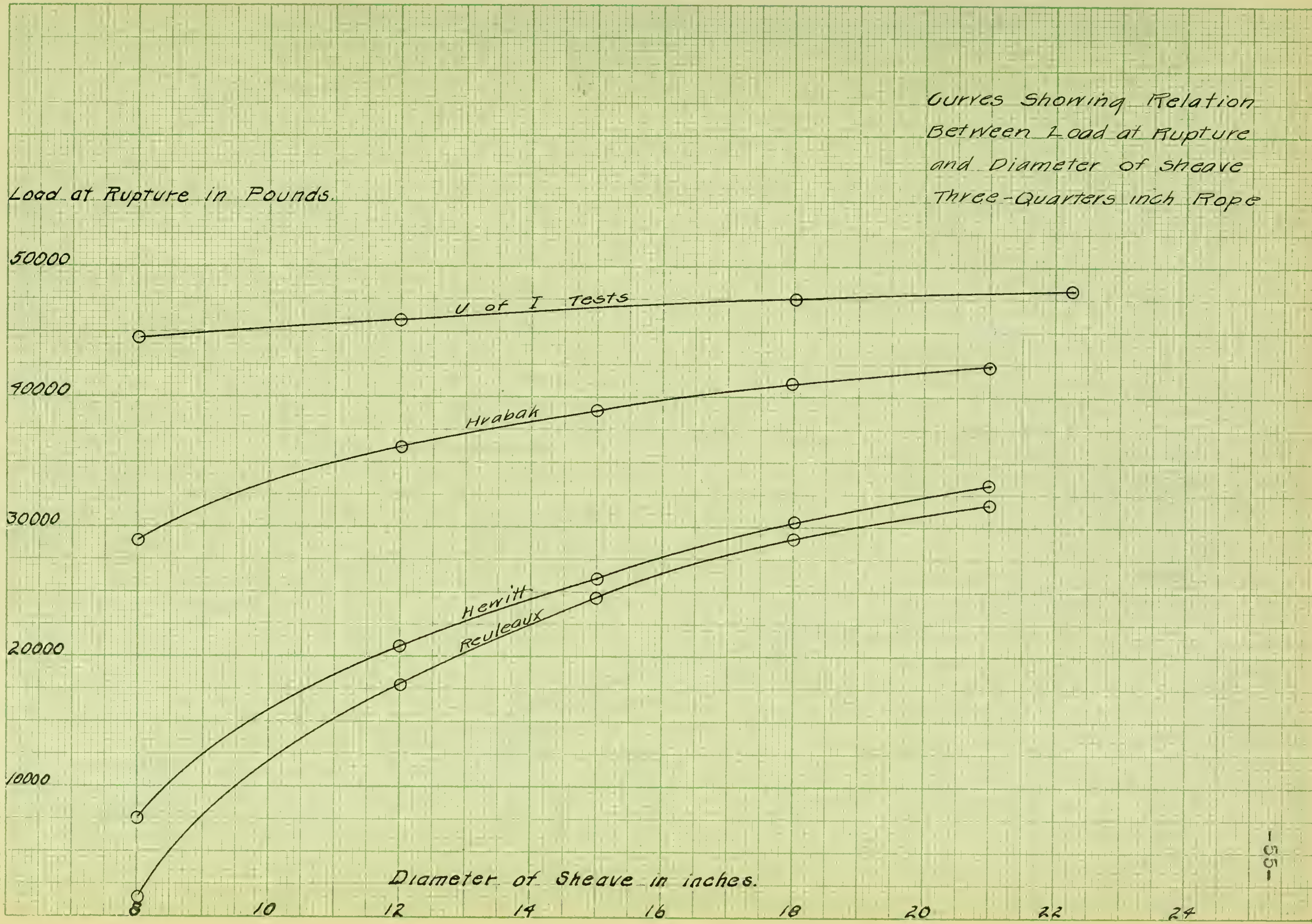
COMPUTED FROM THE VARIOUS FORMULAE

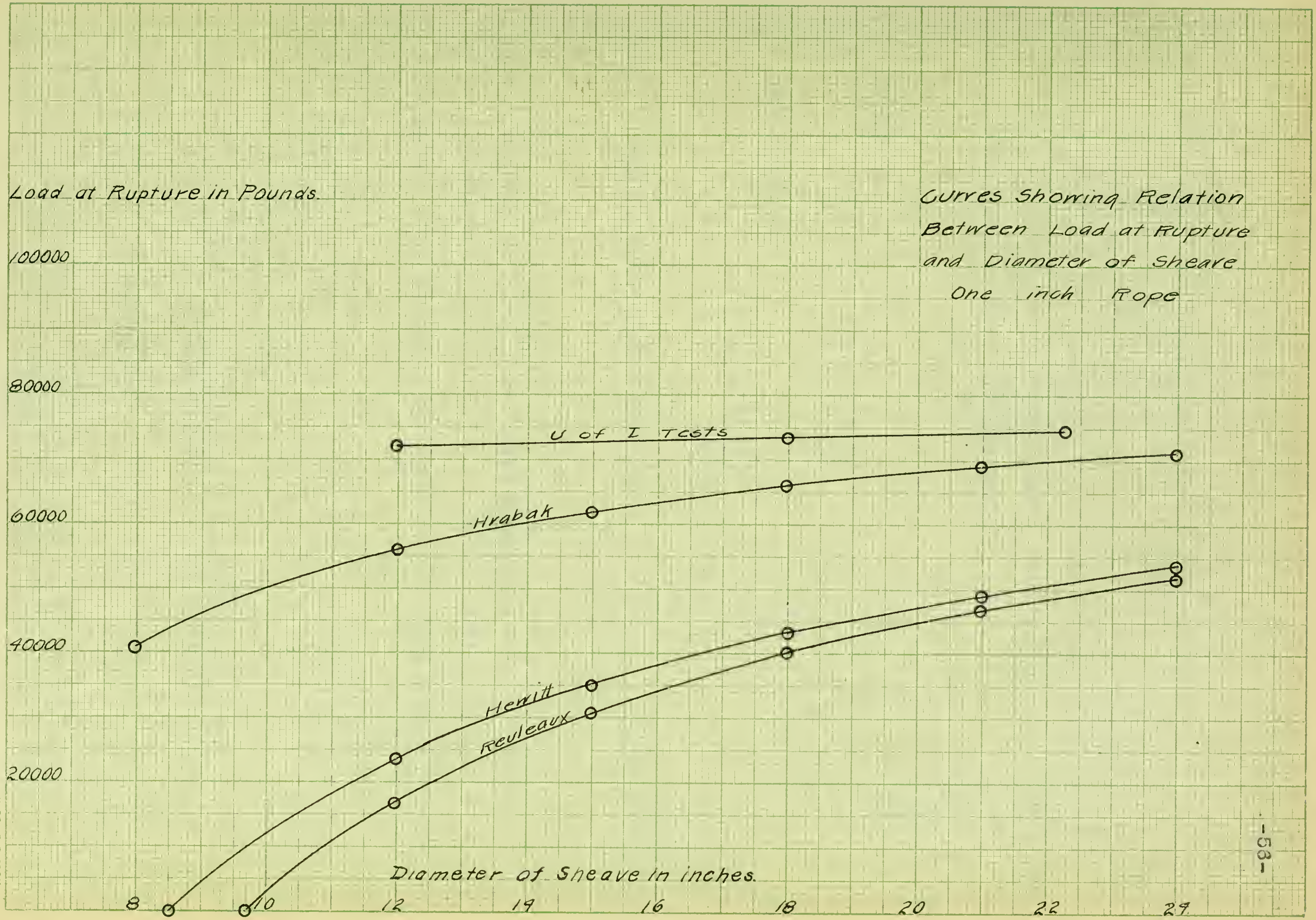

Curves Showing the Relation
Between Bending Stress in Pounds
And Diam of Sheave in Inches
One-half inch Rope
Bending
Stress in
Pounds.
14000
12000
10000
8000
6000
4000
2000
Reuleaux
Hewitt
Hrabak
U of I Tests
Diameter of Sheave in inches
8
10
12
14
16
18
20
22
24


Bending Stress In Pounds.
Curves Showing the Relation
Between Bending Stress in Pounds
And Diam of Sheave in Inches
Three-quarter inch Rope
50000
40000
30000
20000
10000
Reuleaux
Hewitt
Hrabak
U of I Tests
Diameter of Sheave In Inches.
8
10
12
14
16
18
20
22
24


Curves Showing the Relation
Between Bending Stress in Inches
And Diam. of Sheave in Inches
One - Inch Rope.
Bending Stress in Pounds.
120000
100000
80000
60000
40000
20000
Reuleaux
Hewitt
Hrabak
U of I Tests
Diameter of Sheave in inches.
8
10
12
14
16
18
20
22
24


Curves Showing the Relation
Between Load at Rupture and
Diameter of Sheave
One-Half inch Rope
Load at Rupture in Pounds.
20000
16000
12000
8000
4000
U of I Tests
Hrabak
Hewitt
Reuleaux
Diameter of Sheave in inches.
8
10
12
14
16
18
20
22


Curves Showing Relation
Between Load at Rupture
and Diameter of Sheave
Three-Quarters inch Rope
Load at Rupture in Pounds.
50000
40000
30000
20000
10000
U of I Tests
Hvabak
Hewitt
Reuleaux
Diameter of Sheave in inches.
8
10
12
14
16
18
20
22
24


Curves Showing Relation
Between Load at Rupture
and Diameter of Sheave
One inch Rope
Load at Rupture in Pounds.
100000
80000
60000
40000
20000
U of I Tests
Hrabak
Hewitt
Reuleaux
Diameter of Sheave in inches.
8
10
12
14
16
18
20
22
24

IV. DISCUSSION OF THEORIES AND RESULTS.

15. In the introduction to this thesis there are given six distinct formulae for computing the stress in wire rope due to bending. If these formula are reduced to the same units there are really only three. They are Reuleaux's

$$S = \frac{Ed}{D}$$

Hrabak's

$$S = .44\ \frac{Ed}{D}$$

and Chapman's

$$S = .81\ \frac{Ed}{D}$$

16. Hewitt's unwieldy formula $K = \dfrac{Ea}{2.06\ \dfrac{R}{d'} + C}$ reduces to approximately $S = .97\ \dfrac{Ed}{D}$ as below taking 19 wire strands $d' = \dfrac{1}{15}$ diameter of rope and C = ;5.45 and taking case of 1" rope 24" sheave. As K is given in pounds divide each side by a and

$$S = \frac{E}{2.06\ \dfrac{R}{d} + 15.45} = \frac{Ed}{2.06\ R + 15.45d}$$

$$2.06R = 1.03D$$

$$\text{or } S = \frac{Ed}{1.03D + 1.03} = \frac{Ed}{1.03\ (D + 1)}$$

$$\text{or approx. } .97\ \frac{Ed}{D}$$

17. Sederholm's formula $S = 1,894,000 \frac{d}{D}$

where d is the diameter of the rope is just another way of stating the principle developed by Reuleaux.

If 19 wire rope is taken, as this is derived for d = 15d'

$$\text{and } S = \frac{1,894,000 \times 15d'}{D} = \frac{28,500,000d'}{D} \text{ or } S = \frac{Ed'}{D}$$

18. The formula given out by the American Steel and Wire Co. is precisely the same as that given by Hrabak but expressed differently. They say $S = 12,000,000 \frac{d'}{D}$

$$12,000,000 = .44 \times 28,500,000$$

or $S = \frac{.44\ d'}{D}$ which is that given by Hrabak.

19. The writers will therefore only consider the three formulae as stated in the beginning of this section.

Reuleaux developed his expression by taking a rope as a bundle of single wires and finding the stress in the extreme fiber in a single wire. He did this by considering the wire as a simple beam under flexure. From the start the stresses figured in this manner would seem to high because the very object of the spiral construction is to give flexibility. Again by computation from the formula the bending stress of a three-quarters inch rope around a seven inch sheave would be more than enough to cause rupture while every day in practice three-quarter inch ropes are bent around two inch diameters in making thimble grips. And finally this equation would be in error mathematically because it does not take into consideration the angle of lay of the

wires and strands and the direction of lays.

20. Hrabak developed his theory partially by experiment. He found that a built up rope stretched about three times as much as if made up of a bundle of parallel wires. His expression for this result was $E = .36\ E_o$

He then tried to take into account the angle of lay and by an elaborate mathematical investigation found that

$E_1 = E \sec^2 a \ \ \sec^2 b$. and taking the angles of lay as 18°

$$E_1 = 1.222\ E = .44\ E_o$$

$$\text{or } S = .44\ \frac{Ed}{D}$$

From first principles of mechanics there appears to be no reason why the modulus of elasticity of the wires as laid on the rope should be different from that of the same wires when laid parallel. It is true of course that the stretch will be different when the wires are laid up in a rope due to the tendency to untwist and the wires to straighten out. It is equally true as has been shown by experiment that the stretch in an old wire will only be about half as great as that in a new wire and no doubt for old ropes long in service Hrabak would have found E equal to about $.75\ E_o$. Aside from the question of modulus of elasticity Hrabak's theory seems to be open to question in another direction. In correcting his preliminary result for lays of wires and strands he gets an increase of stress due to flexibility which on the face of it seems absurd.

21. Mr. R. W. Chapman in a theoretical investigation of the stress in wire ropes due to bending finds that $S = .81 \frac{Ed}{D}$

He uses a proposition of advanced geometry according to which the radius of curvature of any normal section of a surface is given by the formula

$$\frac{1}{r} = \frac{\sin^2 a}{R_1} + \frac{\cos^2 a}{R_2}$$

Where R_1 and R_2 are the radius of curvature of the two principal normal sections and a is the angle which the section whose radius is r makes with the first principal normal section.

By applying the formula to the case of wire rope he finds that

$$S = \frac{Ed}{D} \cos^2 a \cos^2 b$$

when a and b = 18°

$$S = .81 \frac{Ed}{D}$$

22. The results of the tests, as can be seen by the curves, show very much lower stresses due to bending than those computed from the formulae. Indeed the results would seem to show that the stress due to bending was almost negligible and that the ruptures were lower for the small sizes of sheaves due to the greater cutting action and the combination of forces at the rupture point. An examination of the curves of individual tests show that the yeild points of the straight and curve sections are practically alike in all cases, emphasizing more strongly the

small amount that the bending stress is in relation to the whole. A further inspection of these curves show that there are in general three yield points (See Test No. 4) due no doubt to the fact that in a given length of strand there are three lengths of wires, that of the center, that of the six wires around it, and that of the twelve outside wires. It is probable that the center wire takes a considerable portion of the load first and as it elongates the others take the stress in turn, thus giving three distinct yield points in cases where the instruments were left on very close to the rupture point.

23. It can be seen by an examination of data that the cases where four strands broke showed higher ruptures than the cases where three strands broke. Coupling this with the fact that the strands next to the sheave ruptured in all cases, it is evident that the rupture depends also on the lateral compression the strands receive between the upper strands, having a compound force toward the center of the sheave, and the reaction of the sheave against the lower strands.

24. The matter of nicking was very serious in all of the specimens tested even in the straight pull tests. While it is true that the nicking will not be so severe with the loads applied to the rope in practice, it is also true that the nicking will be there to some extent and would be an important factor in the wear and life of the rope.

25. From previous investigations and from these tests the writers find that the static stress in a wire rope due to bending

15

30 00

450 00

depends upon the following factors:

Diameter of single wires

Diameter of strands

Diameter of rope

Diameter of sheave

Angles of lay

Modulus of elasticity

Number of wires

Number of strands

The friction between the wires or its stiffness

Speed of bending

Weight of rope if centrifugal force acts.

Length of wires in a length of strand.

26. It seems to the writers that all of these factors enter into the stress of bending and that it would be impossible without an elaborate series of tests covering many years to determine the exact effect of each and to derive an equation mathematically correct which would include all of these factors. They believe that a series of tests of various types of rope of various sizes of sheaves should be made and an empirical formula derived to cover the ground in the best manner possible. With the tests already made this would be $S = .15 \frac{Sd}{D}$

27. Wear is an important factor in the selection of the proper size of sheave for any size rope as the curve by Mr. Biggart shows. The writers believe that the subject of wear is much more important than the subject of static bending stress

and that the tests of the future should be made to determine more accurately its effect.

IV. CONCLUSIONS.

1. The bending stress in a wire rope is much smaller than had ordinarily been assumed.
2. The proper size of sheave for a size of rope should be calculated more for wear than for bending stress.
3. The cutting action of one wire upon another is very severe and is a large factor in the wear and life of a rope.
4. The empirical formula for bending stress as calculated from the results of these tests is $.15 \frac{Ed}{D}$.